The parameter was chosen so that the effect on $L$ is given by
\[
\hat{L}' = (p^2 - m_1^2 + m_2^2) \Gamma' - 2m_2 \Gamma' \rho_0 - \gamma.
\]
This is the same form as $L$, with $m_1$ and $m_2$ interchanged, which reveals a symmetry of the wave operator, of the solutions of the wave equation, and ultimately of $T$. To express this symmetry in terms of the momentum variables we must define $q''$ symmetrically, thus, while $q'' = m_1 \Gamma' \rho_0 \rho_1$, we must define $q'' = m_2 \Gamma' \rho_0 \rho_1$. Then
\[
T(p, q, m_1, m_2) = T(p, q', m_2, m_1).
\]
Here a single set of momentum variables represents both initial and final values, to avoid a cluttered notation.

The expression for $q'$ in terms of $q$ is
\[
q'' = m_1 \left( \frac{p_0 \cosh\theta + m_1 \sinh\theta}{m_1 \cosh\theta + q_0 \sinh\theta} \right),
\]
\[
q' = m_1 \frac{-m_0 q}{m_1 \cosh\theta + q_0 \sinh\theta}.
\]

On the mass shell
\[
(p-g)^2 = m_2^2
\]
or
\[
q_0 = (p^2 + m_1^2 - m_2^2)/2p_0,
\]
which gives
\[
q' = (p^2 - m_1^2 - m_2^2)/2p_0 = p_0 - q_0,
\]
and $q' = -q$. Thus, on the mass shell $q' = p - q = p_2$, and the symmetry of the $T$ matrix takes the form
\[
T(p, p_1, m_1, m_2) = T(p, p_3, m_2, m_1),
\]
or, briefly, since $p = p_1 + p_2$ and $p_1^2 = m_1^2$, $p_2^2 = m_2^2$,
\[
T(p_1, p_2) = T(p_3, p_4),
\]
which is the desired result.

So far we have treated $\gamma = \Gamma V$ as a constant. A sufficient condition for the symmetry of the scattering matrix is that $\gamma$ be symmetric (invariant) under the above transformation, including the interchange of $m_1$ and $m_2$. This condition is not necessary, since $V$ is not completely determined by the on-shell $T$ matrix. A necessary condition is that $V$ be symmetric on shell.

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**Feynman-Like Diagrams Compatible with Duality. II. General Discussion Including Nonplanar Diagrams**

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A general formulation of duality theory is presented that includes nonplanar Feynman-like diagrams. All diagrams, planar as well as nonplanar, are so classified that the diagrams in a given class are mutually connected by duality. A prescription is given for constructing an integral representation of the scattering amplitude for each class. Some fundamental properties of the duality relations are discussed.

**I. INTRODUCTION**

In Paper I we have discussed planar Feynman-like diagrams (FLD), the corresponding duality amplitudes, and their high-energy behavior. In this paper we continue the program to include nonplanar diagrams.

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In the case of the planar diagram, all the FLD's with \( n \) external lines and \( L \) loops are connected by duality, and the corresponding dual diagrams are obtained by various triangulations from a duality diagram which is an \( n \)-sided polygon with \( L \) points inside it. Thus planar duality amplitudes are specified by \((n, L)\), and an integral representation can be written for each \((n, L)\) using the prescription given in I. It is interesting to note that the sides of the polygon correspond to the external lines of the FLD, and the inside of the \( n \)-sided polygon maps onto the surface of a sphere with an \( n \)-sided window taken out.

There are two questions to be answered in order to extend the program to include nonplanar diagrams. (i) Can one exhaust all the possibilities of duality relations by representing them in terms of FLD's which are composed of lines and vertices alone? (ii) Can the duality amplitudes of nonplanar diagrams be classified by \((n, L)\) alone? If not, how can one classify them? These will be answered in Sec. II., and a complete classification of FLD's will be obtained by the classification of duality diagrams, for which we shall use elementary topology on two-dimensional surfaces.

In Sec. III we write down the integral representation of the duality amplitudes using the duality diagram.

The final section is devoted to the discussions on miscellaneous problems, mostly of a technical nature.

II. CLASSIFICATION OF FEYNMAN-LIKE DIAGRAMS

A. Definition of Feynman-Like Diagrams

In order to answer the first question of the previous section, let us consider the duality relation expressed in terms of the quark model, in which the twisted propagator such as Fig. 1(A) should be distinguished from the untwisted one such as Fig. 1(a). Therefore the usual Feynman diagrams expressed in terms of lines and vertices are not sufficient to exhaust all the possibilities. (See Sec. IV for further discussions of this subject.) In the following we express the two ways of exchanging a tower of resonances in \((a) \rightarrow (b)\) and in \((A) \rightarrow (B)\), respectively, by the untwisted line [Figs. 1(a)'] and 1(b')] and the twisted line [Figs. 1(A)'] and 1(B')]. We call the connected diagram composed of these lines and three-point vertices a Feynman-like diagram (FLD). One can, however, express the twisted line exchange in Fig. 1(A) or 1(A) in another way (see Fig. 2). Therefore, the following shifting rules for the twisting cross should be kept in mind in drawing FLD's.

(i) A twisting cross on a line can be distributed to the other joining lines by changing simultaneously the order of the lines in the vertex (see Fig. 2).

(ii) A twisting cross on an external line can be removed.

(iii) Two twisting crosses on a single internal line can be eliminated.

As a consequence of the shifting rule, all the FLD's can be classified into two large classes, \((O)\) and \((NO)\). Class \((O)\) can be drawn without using twisted lines; if all the twisting crosses can be removed from a given FLD by successive applications of the shifting procedure, the diagram is considered to belong to this class. Class \((NO)\) consists of diagrams which can not be drawn unless the twisted line is used. An example of each is given in Fig. 3. We call the former the “orientable class” \((O)\) and the latter the “nonorientable class” \((NO)\). The meaning of the terminology will be clarified later.

B. Dual Diagrams

Once an FLD is given, orientable or nonorientable, the corresponding dual diagram can be drawn in the following way. (i) Make triangles each of which is associated with a vertex in the FLD. In making the triangle the order of the sides should be arranged in

\[\text{Fig. 2. Another way of expressing the twisted diagram, and the shifting rule of twisting crosses.}\]

\[\begin{align*}
\text{(a)} & \quad \Rightarrow \quad \text{(b)} & \quad \Rightarrow \quad \text{(a)}' \\
\text{(A)} & \quad \Rightarrow \quad \text{(B)} & \quad \Rightarrow \quad \text{(A)}' \\
\text{(a)} & \quad \Rightarrow \quad \text{(b)} & \quad \Rightarrow \quad \text{(a)}' \\
\text{(A)} & \quad \Rightarrow \quad \text{(B)} & \quad \Rightarrow \quad \text{(A)}'
\end{align*}\]
Fig. 3. (O) Various ways of expressing an orientable diagram; (NO) various ways of expressing a non-orientable diagram.

Fig. 4. Cut-and-glue processes of making dual diagrams; (a) corresponds to Fig. 3(O), and (b) to Fig. 3(NO).
according to the order of joining lines in the corresponding Feynman vertex. (ii) Next, connect these triangles together in such a way that they share a side if two triangles have the same side in common. In connecting them do not flip the triangle over if the relevant sides are associated with an untwisted line in the FLD, and do flip one of the relevant triangles if the sides are associated with a twisted line in the FLD.

As a consequence we will end up with a two-dimensional surface with boundaries. The boundaries correspond to the external lines. The surface is triangulated in a certain way according to the structure of the FLD. It is easy to see that thus-constructed dual diagrams have a one-to-one correspondence to FLD’s. From the construction of the dual diagrams it is clear that the dual FLD’s of the orientable class are on orientable surfaces while the others on nonorientable surfaces.3

As examples, the dual diagrams corresponding to the FLD’s in Fig. 3 are drawn in Fig. 4, from which it is obvious that the shifting rule of the twisting crosses is nothing but the cut-and-glue rule of equivalent two-dimensional surfaces.3

C. Classification of FLD’s

We classify the dual diagrams into classes in each of which the corresponding FLD’s are connected with each other by duality.

Let us begin with a vacuum fluctuation diagram which has no external lines (n = 0). The corresponding dual diagram should form a closed surface. Thus the classification of n = 0 diagrams is reduced to that of closed surfaces. We quote here a well-known theorem:4

Any compact surface is either homeomorphic to a sphere or to a connected sum of tori, or to a connected sum of projection planes.

The former two are called “orientable” surfaces and the latter “nonorientable surface.” Thus each closed surface is specified by the orientability ε (+1 for orientable and −1 for nonorientable) and an integer p, which represents the number of tori or the number of projection planes depending on whether ε = +1 or −1, respectively. In terms of the standard symbols, a sphere (ε = +1, p = 0), a torus (ε = +1, p = 1), and a projection plane are described, respectively, as

\[ aa^{-1}, \quad aba^{-1}b^{-1}, \quad \text{and} \quad aa. \]  

(2.1)

The canonical polygon representations of these are given in Fig. 5.

The triangulation of a surface is done by first drawing points on the surface and then connecting them by lines appropriately so as to have triangles alone on the surface. As in I, the number of points on a given surface is related to the number of loops L of the corresponding FLD. Some of the triangulations of the surfaces (2.1) to obtain the dual diagrams and the corresponding FLD’s are demonstrated in Figs. 6–8, which do not need further explanation.

As a consequence of the previous discussion, it is clear that for n = 0 we can specify the classes of dual diagrams [or, equivalently, duality diagrams] by (ε, p, L) such that any two of the same class are connected by duality. We denote these classes by \( C^{(2)}(\epsilon, p; 0) \), where 0 refers to n = 0 [see Eq. (2.2)].

The dual diagram associated with an FLD which has some external lines is easily obtained once we notice that the external lines should correspond to the boundary edges of a compact surface. A two-particle scattering diagram (four external lines) can be obtained by cutting out of a closed surface a window. The boundary frames of which are composed of the four external lines. In Fig. 6(a) or 6(b) we take out two triangles (1, 5, 2) and (4, 3, 5) (shaded area) to get a scattering 1 + 4 \rightarrow 6 → 2 + 3, which is visualized by erasing the part surrounded by the dashed line of Fig. 6(c). In Fig. 7(a) or

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7(b) and Fig. 8(b), likewise, taking off (1, 5, 4) and (3, 2, 5) we get FLD’s which are obtained by erasing the part surrounded by the dashed line in Fig. 7(c) and by Fig. 8(c). One-window diagrams [some of them are Figs. 6–8(b)] are in general written as Fig. 9(a). Another four-point dual diagram is obtained by taking two windows out of a closed surface, both with two frames or one with a frame and the other with three frames [Figs. 9(b) and 9(c)]. One can make up to four windows in which each window is made of a single frame [Figs. 9(d) and 9(e)]. These five diagrams are all topologically independent so that they provide us with different duality diagrams, and, accordingly, with duality amplitudes.

In general, for the \( n \)-point amplitudes we obtain all the surfaces of dual diagrams by taking up to \( n \) windows out of the closed surfaces. Because of the following theorem, these surfaces exhaust all the possibilities:

Any compact surface with boundaries is either homeomorphic to a sphere with \( w \) windows, or to a connected sum of tori with \( w \) windows, or to a connected sum of projection planes with \( w \) windows, where \( w=1, 2, \ldots \).

The \( n \)-point FLD’s with \( L \) independent Feynman loops are classified into classes which are specified by

\[
C_n^{(L)}(\epsilon; \rho; w, \delta),
\]

where \( w (=1, 2, \ldots, n) \) represents the number of windows taken out of the surface \( (\epsilon, \rho) \), and \( \delta \) indicates the way of distributing the \( n \) external lines to the \( w \) windows.

### III. INTEGRAL REPRESENTATIONS

We introduce the "duality diagram" to the class \( C_n^{(L)}(\epsilon; \rho; w, \delta) \), a generalization of the same terminology introduced in I for planar classes. It is obtained by erasing all the internal lines from a given dual diagram which belongs to this class, but keeping the vertex points and boundary edges unerased. Examples are shown in Fig. 9, where one can add an arbitrary number of points inside the polygons. The diagram so obtained has the same form for all the dual diagrams of the class \( C_n^{(L)}(\epsilon; \rho; w, \delta) \). In other words, all the dual diagrams which belong to this class are obtained by various triangulations of the duality diagram. It is easy to see that the number of internal...
lines \( m \) is invariant so that the number of triangles \( f \) is also:
\[
3f = 2m + n. \tag{3.1}
\]

Each line carries its momentum, which can be read off successively by applying momentum conservation to each triangle. So any line momentum can be written as a linear function of the external momenta \( p_i \) (\( i = 1 \cdots n \)) and the loop momenta \( k_i \) (\( i = 1 \cdots L \)), since the position of all points in the duality diagram is fixed if we fix \( p_i \) (\( i = 1 \cdots n \)) and \( k_i \) (\( i = 1 \cdots L \)). Let \( \nu \) be the number of points in the dual diagram; \( n \) points out of \( \nu \) are used to make boundaries. The relative location of windows should be fixed to fix the boundaries; note that \( w - 1 \) loop momenta are necessary to do this. Since \( \nu - n \) points are left unfixed inside of the boundary, an additional \( \nu - n \) loop momenta are necessary to fix them all. In order to fix the form of the surface completely, we must fix the distance of pairs of points in the polygon representation, each of which pairs represents the same point of the surface. This requires additional \( 2 - \chi \) loop momenta, where \( \chi \) is the Euler characteristic of the closed surface and is given by
\[
\chi = 2 - 2p \quad \text{for } e = +1
\]
\[
= 2 - \rho \quad \text{for } e = -1. \tag{3.2}
\]

Thus,
\[
L = \nu - n + w - 1 + 2 - \chi. \tag{3.3}
\]

Since the number of lines (internal and external) is given by \( e = m + n \) and since \( \chi \) is given by
\[
\chi = \nu - e + f + w,
\]
we obtain
\[
m = 3(L - 1) + n. \tag{3.4}
\]

To construct the integral representation we first choose \( m \) independent lines and the corresponding integration variables \( x_1 \cdots x_m \). Any other internal line crosses at least one of these \( m \) lines. As in I, the variable associated with this line can be obtained as a function of \( x_1 \cdots x_m \) by repeatedly applying formulas (3.2) and (3.3) of I. Let us denote the lines from the point \( r \) and the point \( q \) in the dual diagram by \( (r, q; \nu) \), where \( \nu \) classifies all possible such lines. Let \( p_{\nu}^{(o)} \) be the momentum carried on this line, \( o_{\alpha}^{(o)} \) the linear Regge trajectory function of this channel, and \( f_{\nu}^{(o)}(x_1 \cdots x_m) \) the variable associated with this line.

To determine \( p_{\nu}^{(o)} \), we first take the origin of momentum space on the duality diagram at a point which is, say, on a boundary edge of one of the windows. Next we associate the loop momenta \( k_i \) (\( i = 1, 2, \ldots, L \)) with \( L \) lines which connect independent points of the polygon representation of the duality diagram in such a way that these \( L \) lines together with \( n \) external lines do not form any contractible closed path.\(^3\) The remaining \( n - w \) points are on boundary frames of windows whose corresponding 4-vectors can be determined by the vector sum of \( k \)'s and the known external momenta, because at least one and only one of the points which lie on the boundary frame has to be an end point of the \( L \) independent lines. The vertex so determined is directed; if the distance from \( q \) to \( r \) is \( k \), that from \( r \) to \( q \) is \( -k \). All other vectors from a vertex to others can be measured along those \( L \) specified lines and/or external lines. One should note that the 4-vector which corresponds to a closed path (a line starting from a point and returning to the same point) is not necessarily zero. Some examples are shown in Fig. 10. It should be also noted that the direction of momentum can be matched to the direction of the gluing edges in a polygon representation if the surface is orientable. But, if it is non-orientable, the arrow of the polygon representation does not always agree with that of the momentum; one of a pair of edges of the polygon (not edges of window).
should have one momentum antiparallel to the gluing indication [Fig. 9(c)].

The integral representation of the dual amplitude of $C_n^{(g)}(\rho, \delta; w, \delta)$ is then given by

$$I(C_n^{(g)}(\rho, \delta; w, \delta)) = \int \prod_{i=1}^{n} d^{4}k_{i}$$

$$\times \int_{0}^{1} \prod_{i=1}^{n} \int_{a}^{b} \prod_{r,s,t} \left[ f_{q,r,s}(x_{1}, \ldots, x_{n}) \right]^{-\alpha_{q,r,s}(\rho \delta^{(g)} \delta)} - 1 \times V(x_{1}, \ldots, x_{n}; \alpha_{q,r,s}(\delta)) \tag{3.5}$$

where $V(x_{1}, \ldots, x_{n}; \alpha_{q,r,s}(\delta))$ is a function that does not correspond to any lines (propagators) in the FLD, satisfies certain symmetry properties required from the imposed crossing symmetry, and is otherwise arbitrary.\(^4\)

\(^4\) If one requires the complete factorization of the pole residues, the function $V$ consists of all the line functions which does not, however, correspond to any propagator in the FLD. See K. Bardakci, M. B. Halpern, and J. A. Shapiro, Phys. Rev. 185, 1910 (1969); note added in proof of S. Fujii and G. Veneziano, Nuovo Cimento 88A, 811 (1969); note added in proof of Ref. 1. If one does not require the complete factorization, one has to multiply an invariant volume factor with respect to the crossing-symmetry reflection, which can be obtained by using the standard method as is obtained in the Riemann geometry.

Example: The amplitude of $C_n^{(g)}(+1, 0; 2, \delta(3, 1))$ has the following integral representation, where $\delta(3, 1)$ represents the distribution of external lines, namely, 1, 3, and 4 to one window and 2 to the other window [see Fig. 9(c)]; in this case, $\rho = 0$, it can be conveniently drawn as Fig. 11]:

$$I = \int \prod_{i=1}^{4} \int_{0}^{1} d^{4}k_{i} \times V(x_{1}, \ldots, x_{4}; x_{1} - \alpha(k_{4}) - 1, x_{2} - \alpha(k_{3}) - 1, x_{3} - \alpha(k_{2}) - 1)$$

$$\times x_{0} - \alpha(k_{4} + k_{3} + k_{2} - 1, k_{1} - \alpha(k_{2}) - 1)$$

$$\times x_{y} - \alpha(k_{4} + k_{3} + k_{2} - 1, k_{1} - \alpha(k_{2}) - 1)$$

$$\times x_{y} - \alpha(k_{4} + k_{3} + k_{2} - 1, k_{1} - \alpha(k_{2}) - 1) \tag{3.6}$$

where $V(x_{1}, \ldots, x_{4})$ is a symmetric function under $x_{1} \rightarrow x_{2}$ and $x_{4} \rightarrow x_{3}$. The dependent lines $y_{i}$ are given by $y_{1} = f(x_{5}, 0x_{6}x_{5})$

$$= (1 - x_{5})/(1 - x_{5}x_{1} - x_{5}x_{4}) \frac{1}{x_{1}}$$

$$y_{2} = f(x_{5}, 0x_{6}0x_{5}) = (1 - x_{5})/(1 - x_{5}x_{2})x_{5}$$

$$y_{3} = f(x_{5}, 0x_{6}x_{5}0) = (1 - x_{5})(1 - x_{5}x_{3}x_{5})/(1 - x_{5}x_{5})$$

$$y_{4} = f(x_{5}, 0x_{6}x_{5}0) = (1 - x_{5})(1 - x_{5}x_{4}x_{5})/(1 - x_{5}x_{5}) \frac{1}{x_{4}}.$$
To determine momenta which are carried by lines, we have taken 0 as the origin of momentum space and the point $A(\mathbf{x}_1)$ is specified by the momentum $k$. Then the momentum $P_i$ carried by the line $x_i$, for example, is given by $p_i + p_A = k$. The momentum carried by $y_i$ is given by $P_i + p_0$, and so on. One may add an arbitrary number of other dependent lines to Fig. 11.

Another interesting example of the integral formula and its high-energy behavior has been discussed in Ref. 5.

IV. MISCELLANEOUS PROBLEMS

This section is devoted to further studies on the duality relations among various FLD's.

A. Crossing Symmetry

First we should like to show that any duality amplitude constructed by (3.5) corresponding to a class $C^{(L)}(s, t; \mu, \delta)$ can be made $s \rightarrow t$ symmetric by a suitable arrangement of external lines. The $s \rightarrow t$ crossing symmetry of 4-point functions may be represented by an interchange of external lines 1 and 3 in Fig. 12.

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Fig. 11. (a) All lines contained in (3.6); (b) independent internal lines $x_i$ and their momenta.

Fig. 12. Crossing reflection for $s \rightarrow t$.

Fig. 13. Deformation of FLD's by duality; (a), (b), and (c), and (d), (e), and (f) are connected mutually by duality.

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Fig. 14. Method of getting the quark-model graph (b) from the corresponding duality diagram (a).

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*K. Kikkawa, Phys. Rev. 187, 2249 (1969).* The dual diagram which has been given in this reference is different from that given in the present text, but the former is of course topologically equivalent to the latter.
keeping 2 and 4 invariant. In the duality diagram this symmetry will be shown in the following way. Let us draw a canonical polygon representation of duality diagrams in such a way that the polygon is symmetric with respect to a vertical line, say OO' in Fig. 9, where the outer edge of the polygon $a_i$ (not the edge of windows) may be mapped to $a_i$ under this reflection. Furthermore, let us so arrange the external lines that 1 and 3 are mapped onto 3 and 1, respectively, and 2 and 4 are mapped onto themselves, as is shown in Fig. 9 (a particular choice of the index $\delta$). Then, since the crossing symmetry is guaranteed by the invariance of the amplitude under the reflection with respect to OO', our amplitude is invariant if we insert lines $f_{\mu,\nu}(\theta)$ in the duality amplitude in such a way that the diagram keeps the reflection invariance. In other words, an FLD is always connected by duality with the FLD which can be obtained from the former by an interchange $1\leftrightarrow 3$.

The other amplitudes which have a different index $\delta$ from those of Fig. 9 are obtained by interchanging external lines or by interchanging the variables $(s, t)$ to $(t, u)$ or $(u, s)$ in the above amplitude. The most general amplitude is, therefore, given by a linear combination of $V(s, t)$, $V(t, u)$, and $V(u, s)$, where $V(x, y) = V(y, x)$.

B. Orientable or Nonorientable Diagrams

The necessity (or non-necessity) of the nonorientable FLD's cannot be determined without introducing the dynamics. If one confines oneself to meson interactions in the quark model, nonorientable diagrams will be prohibited because the loop lines in Fig. 3 (NO), say, represent a triality-nonzero object. But if one takes baryon states into account, nonorientable FLD's are allowed because the loop may be identified with a baryon loop by adding an additional quark line as is shown in Fig. 3 (NO) by a dashed line.

C. Duality Connection of FLD's

If one FLD is connected by duality to another, the latter can be obtained from the former by the following simple rule. Let us suppose that each line in the FLD is made of a ribbon (or tape). If the line has a twisting cross, the ribbon is twisted once; if the line has no twisting cross, the ribbon is likewise untwisted. The application of the duality rule (Fig. 1) generates a new diagram which is obtained by sliding an attached point of a line (ribbon) along the edge of another ribbon [Fig. 13(a)]. When a sliding line encounters a twisted cross, it goes to the opposite side, with a twist in the line [Fig. 13(d)]. When two (or more) lines, say, 1 and 2 in Fig. 13, meet at a point on the edge of ribbon, 1 (or 2) can climb up the edge of 2 (or 1) [Figs. 13(c) and 13(f)].

D. Relation to Quark-Model Graphs

In order to visualize the duality relations, Harari and Rosner\textsuperscript{4} have used a quark-model graph. Such a graph can be considered as an intermediate between the duality diagram shown in Figs. 6--9 and the FLD. The quark-model graph can be obtained either by replacing a line in an FLD with a ribbon as in Fig. 3 or by replacing a vertex point in a dual diagram with a hole. To show the latter method, we give some examples in the following.

In a planar diagram with one loop shown in Fig. 14(a), we cut off some parts in such a way that all vertex points are removed, and then enlarge the obtained hole to get Fig. 14(b), which is identified with the quark-model graph.

In the case of the nonplanar diagram, for example, Fig. 7, we cut off the four corners of the window (in this case these corners are all the vertex points on the torus) to get Fig. 15(a). Enlarging the window, we obtain Fig. 15(b), which is further deformed until Fig. 15(c) without changing the topological structure of the surface.

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