EVIDENCE AGAINST NARROW-BAND SPATIAL FREQUENCY CHANNELS IN HUMAN VISION: THE DETECTABILITY OF FREQUENCY MODULATED GRATINGS

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INTRODUCTION

It is well known that the ear can "hear out" the spectral components of a complex sound. Recently it has been suggested that the human visual system contains analogous mechanisms that are tuned to rather limited ranges of spatial frequencies (Campbell and Robson, 1968; Sachs, Nachmias and Robson, 1971). These studies typically employ sinusoidal gratings—patterns of stripes in which the luminance along one direction varies sinusoidally. Spatial frequency is the number of stripes per degree of visual angle.

Evidence for narrow-band spatial frequency mechanisms is provided by studies in which detection of a sinusoidal grating is facilitated by a second, near-threshold grating of variable spatial frequency (Sachs et al., 1971; Kulikowski and King-Smith, 1973; Lange, Sigel and Stecher, 1973). The test grating is facilitated only by gratings of rather similar frequency. The bandwidth of a channel tuned to a given frequency may be expressed in terms of the degree to which the second grating, of different frequency, facilitates detection of the test pattern. The bandwidth of the channel centered near 14 c/deg seems indeed sharp: sensitivity falls to a value about 33 per cent of the maximal sensitivity according to measurements by Kulikowski and King-Smith (1973) and King-Smith and Kulikowski (1975). [Other data suggest that channel bandwidth may be broader, on a logarithmic scale, at lower frequencies. Sachs et al. (1971) showed that channels below 2.8 c/deg were broader than the channel measured at 14 c/deg. Quick (1973) concludes, on the basis of measurements near 5, 7, 14 and 21 c/deg, that channel bandwidth on an octave scale may decrease linearly with increasing frequency.]

Evidence for narrow-band channels, whose bandwidths are independent of spatial frequency, can be extracted from a study by Hoekstra, van der Goot, van den Brink and Bilsen (1974) who measured the threshold of a sinusoidal grating as a function of spatial frequency (1 c/deg to 7 c/deg), luminance, and most importantly, the number of cycles. The threshold decreased as the number of cycles increased until a saturation point was reached. As luminance increased from 2 to 165 cd/m², the number of cycles for saturation increased from 3 to 8. The results can be interpreted (Appendix C) as evidence for the existence of mechanisms whose bandwidths are even narrower than those observed by Kulikowski and King-Smith (1973) for gratings detection.

There is an alternative explanation for these findings which does not require the existence of narrow-band mechanisms. Whenever gratings of different spatial frequencies are added together, there will be certain limited regions where the contrasts add completely; at many other regions, the patterns will tend to cancel each other. The contrasts of the two gratings will add completely only where the two patterns are exactly in phase and the contrasts will subtract where the patterns are out of phase. The patterns are not of a constant contrast but are modulated in amplitude and slightly in frequency—hence, quasi amplitude modulated gratings.

The facilitation of one grating by another is probably not dependent only on the peak contrast value of the compound grating. Possibly, gratings are detected...
The output of this oscillator was sinusoidally frequency modulated, which provided the Z axis signal. The sine-wave modulation voltage was set with a vacuum tube voltmeter, and frequency was set with a digital frequency meter. A second oscilloscope was used to monitor the Z axis signal.

Methods

Apparatus

A vertical grating was generated on a CRT (white P-4 phosphor) following the method of Campbell and Green (1965). The grating was a simple sinusoidal grating or sinusoidally frequency modulated sinusoidal grating. The display field was 6° dia with a black surround, viewed from 81 cm. A fine horizontal and diagonal cross hair was placed across the field to aid focussing.

A Wavetek Model 112 oscillator served as a frequency modulator, which provided the Z axis signal. The sine-wave output of this oscillator was sinusoidally frequency modulated by a second sine-wave generator that fed into the Wavetek oscillator. The Wavetek oscillator was calibrated by reading the output frequency with a digital frequency meter. The relationship between input voltage and output frequency was linear for the range of frequencies in the present experiment. When a sine-wave was fed into the Wavetek oscillator, the peak voltages of the sine-wave input produced a particular maximal deviation of the instantaneous frequency of the output. The range of frequency modulation, Δf, is said to be the maximal deviation of the instantaneous frequency away from the mean frequency. This value was known from the d.c. calibration and will be used to determine the spectrum of the FM gratings.

Contrast of the gratings is conventionally defined

\[ c = \frac{L_{\text{max}} - L_{\text{min}}}{L_{\text{max}} + L_{\text{min}}} \]

Grating modulation voltage was set with a vacuum tube voltmeter, and frequency was set with a digital frequency meter. A second oscilloscope was used to monitor the Z axis signal.

Stimuli

The luminance distribution across a grating may be represented by the general expression

\[ L(x) = L_0[1 + \alpha(x)] \]

where \( \alpha(x) \) is an arbitrary function of space and \( L_0 \) is the mean spatial luminance. The sinusoidal grating in the present experiment is represented by

\[ L(x) = L_0[1 - A \cos(2\pi f x + m \sin 2\pi f_t x)] \]

where \( f \) is spatial frequency and \( A \) is the modulation amplitude. The sinusoidally frequency modulated sinusoidal grating may be represented by

\[ L(x) = L_0[1 - A \cos(2\pi f x + m \sin 2\pi f_t x)] \]

where \( f_t \) is the modulating spatial frequency and \( m \) is the modulation index (see Black, 1953). The modulation index is defined as the range of frequency modulation (i.e. the maximal deviation of instantaneous frequency) divided by the modulating frequency, that is

\[ m = \frac{\Delta f}{f_t} \]

(mathematical notation)

The spectrum consists of a sinusoidal grating at frequency \( f \) plus components spaced symmetrically either side of \( f \) at intervals equal to the modulating frequency \( f_t \). The amplitude of each component is given by \( J_n(m) \), which are Bessel functions of the first kind and order \( n \). The modulation index thus specifies the amplitude of each component.

The FM gratings were centered at either 4.0 or 4.5 c/deg and were modulated by a grating of 1/3 these frequencies, i.e. \( f/f_t = 3 \). The modulation index \( m \) was 1.434. Figure 2 shows the theoretical spectra of these gratings, and Fig. 1 shows the actual gratings. Note that the maximal amplitude of the spectral components is 0.55. A simple sinusoidal grating of equivalent contrast has a much larger spectral amplitude of 1.00.

These test patterns were chosen for two reasons. Medium-band mechanisms can effectively sum the three equal-amplitude central components at different phase positions across the gratings, whereas narrow-band mechanisms poorly sum the components. If narrow-band channels are used to detect gratings, then these FM gratings should be less detectable than a simple sinusoidal grating of the same contrast. (These notions are presented in a quantitative form in the Discussion.) Secondly, the FM gratings are centered near the peak of the phosopic contrast sensitivity function as measured with sine-wave targets (van Nes and Bouman, 1967; Campbell and Robson, 1968; Sachs et al., 1971). It has been

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Fig. 1. The test patterns: a sinusoidal grating and a sinusoidally frequency modulated sinusoidal grating. Both patterns have the same central frequency $f$ and the same contrast. The modulated pattern has a modulation index $\approx 1.434$ and the ratio of the central frequency to the modulating frequency $f/f_m$ is 3. The patterns shown here are several hundred times threshold contrast.
The detectability of frequency modulated gratings

Theoretical spectra of the sinusoidally frequency modulated sinusoidal gratings used in these experiments. The central frequency components of the gratings, \( f \), was either 4.0 c/deg (top spectrum) or 4.5 c/deg (bottom spectrum). The modulation index \( m \) is 1.434, and the ratio of the central frequency to the modulating frequency \( f_m/fo \) is 3. The amplitude of each spectral component is indicated at the end of each line. Positive and negative lines indicate positive and negative cosine components respectively. The sinusoidal gratings of equivalent contrast were centered at 4.0 or 4.5 c/deg with an amplitude of 1.00.

assumed explicitly by Campbell and Robson (1968) and implicitly by Sachs et al. (1971) that the overall contrast sensitivity function represents the envelope of the contrast sensitivity functions of all the independent channels. By placing the simple and FM gratings at the peak of the contrast sensitivity function, we make the central components at 40 or 4.5 c/deg more visible than the sideband components: Thus the FM grating should not be more detectable than the simple grating merely because one of the sideband components affects a more sensitive channel than does the simple grating. This is the reason why the experiment was done with gratings centered near 4-5 c/deg.

The experiment would be more difficult to do with higher frequency gratings, for example at 14 c/deg, for at these frequencies the contrast sensitivity function changes rapidly. Thus the amplitude of each spectral component would have to be adjusted to compensate for the individual's contrast sensitivity function.

Studies on the subthreshold summation of two gratings with similar frequencies (a quasi amplitude modulated pattern) do indeed suggest that there are narrow-band channels near 5 c/deg. Kulikowski and King-Smith (1973) found very little facilitation between a grating of 50 c/deg and a grating of 30 or 40 c/deg, and King-Smith and Kulikowski (1975) found very little facilitation between gratings of 3.6, 4.0 and 4.4 c/deg. Quick (1973) observed that the facilitation effect fell off sharply to about half strength one half cycle away from 5 c/deg. Sachs et al. (1971) and Pantle (1973) found no facilitation between a grating near 5 c/deg and a second grating one octave lower. And Abadi and Kulikowski (1973) found facilitation between a 50 c/deg sinusoidal grating and only those harmonics of a square-wave grating of either 1/0, 1/67 and 50 c/deg that were very close in frequency to the 50 c/deg sinusoid. Hoekstra et al.'s (1974) study on the effect of field size on grating detection can also be interpreted as evidence for very narrow-band channels within the range 1-7 c/deg (see Appendix C). Thus given the evidence for narrow-band channels near 5 c/deg, it is of interest to see whether these same channels can be demonstrated with FM as opposed to quasi AM gratings.

Procedure

The main experiments compared the detectability of a simple sinusoidal grating and an FM grating of equivalent contrast. Additional experiments were also done to demonstrate that the sinusoidal and modulated gratings were indeed near the peak of the subjects' contrast sensitivity function.

Each run consisted of 75 trials where two types of patterns and blanks were randomly intermixed, each event occurring with 1/3 probability. The subject's task was to rate the visibility of stimuli on a 1 to 5 scale where 1 means blank and 5 means definitely a pattern (Egan, Shulman and Greenberg, 1959). The subject was informed after each trial what the stimulus had been.

A pair of patterns was used within a single run. This helped eliminate variability due to sensitivity fluctuations over time. The pairs of patterns were of two types:

1. A simple sinusoidal grating and an FM grating of identical contrasts, both centered at the same spatial frequency.

2. Two sinusoidal gratings of identical contrasts and different spatial frequencies. These were used to measure the subjects' contrast sensitivity function.

The stimuli were presented in a repeating sequence:

1. 4 sec. blank field.
2. 4 sec. tone plus blank field. During this period, S pressed a button which, on 2/3 of the trials, presented the test pattern for 750 msec. Each of the two types of test patterns and blanks occurred with 1/3 probability. The onset and offset of the pattern produced no visible electrical noise.

The subject practiced at the beginning of each run until he felt he was ready to start. The CRT was viewed with the left eye which was carefully refracted. The experiments were run under two luminance levels. Initially the scope luminance was set to 5.0 cd/m² and no artificial pupil was used. The natural pupil is approx 5.5 mm at this luminance level (Flamant, 1948). and thus retinal illuminance was 120 td. The experiments were then repeated with the scope luminance increased to 75 cd/m² (produced by adding d.c. voltage to the Z axis). Subject CFS used an artificial pupil.
of approx 2.3 mm dia (retinal illuminance 310 td), and subject SK used a 3.0 mm pupil (530 td). The purpose of this second condition was to better assure that the contrast sensitivity function peaked near 4–5 c/deg—at the center frequency of the frequency modulated patterns. The size of the artificial pupil was chosen for each subject because it in fact made the peak of the contrast sensitivity function fall near the desired frequency.

Data treatment

An ROC curve was fitted to the confidence ratings obtained in each run. The curve was fitted by a maximum likelihood technique similar to that of Ogilvie and Creelman (1968), but where the signal and noise distributions were assumed to be Gaussian (Dorfman and Alf. 1969). The technique finds the values of parameters which were most likely to have generated the data. The parameters were the criterion levels (between the rating categories), the \( d' \) value of the test pattern, and the slope of the ROC curve. The \( d' \) value is the measure of detectability used for all results. It is explicitly defined to be the horizontal intercept of the ROC curve plotted on double-axis \( z \) score (standard score) paper. To make this clear, imagine that the subject has a 50 per cent chance of correctly identifying a signal (hit rate), then the \( z \) score of the false alarm rate is \(-d'\), or equivalently, the \( z \) score for the variance of the blanks is \( d' \). For example, a 50 per cent hit rate and 84 per cent correct rejection of blanks, gives \( d' = 1 \).

The slope parameter, \( \alpha \), is related to the widths of the signal and noise distributions by
\[
\sigma_s/\sigma_n = 1 + \alpha d'.
\]
The slope parameter, unlike the other parameters, should be fairly constant from run to run (Nachmias and Kocher, 1970). The first half of the data was analyzed with \( z \) free to vary and then all data were analyzed with \( z \) fixed. The average computed value was \( z = 0.25 \), which is in agreement with the value of \( z = 0.25 \) found by Nachmias and Kocher for detecting flashed spots of light. All \( d' \) values in this paper were calculated with \( z = 0.25 \). For all experiments, each of the two types of signals used within a run was compared independently to the blanks, since we were interested in the detectability of that particular signal relative to the blank trials.

RESULTS

Figure 3 shows the detectability \( d' \) of simple sinusoidal gratings, indicated by open circles, compared to the frequency modulated gratings, closed circles. The bars are \( \pm 1.0 \) S.E. For these data, the gratings were centered at 4.0 c/deg; mean luminance was 5.0 cd/m\(^2\); and the natural pupil was used. Both a simple grating and a modulated grating were used in each run, and the contrast of both gratings were identical. Thus the data points for each run are plotted as pairs, with open and closed circles separated horizontally by a tiny amount for clarity.

Figure 4 shows similar data, where the gratings were centered at 4.5 c/deg; the mean luminance was raised to 75 cd/m\(^2\); and the artificial pupil was used.

Table 1 shows the average ratio of detectability \( d' \) of modulated versus simple gratings and \( \pm 1.0 \) S.E. of these averages. A value greater than one indicates that the modulated grating was more detectable than the simple grating. The average ratio is given for each subject separately and for each of the two luminance levels. The overall average is close to one, 0.96 \( \pm 0.08 \).

The FM gratings were chosen so that the central frequency component would be near the peak of the subjects' contrast sensitivity functions and thus more visible than the sideband components: Thus the FM gratings should not be more detectable than the simple gratings merely because one of the sideband components stimulates a more sensitive channel than does...
The detectability of frequency modulated gratings

Table 1. The average ratio of detectability $d$ of modulated versus sinusoidal gratings and $\pm 1.0$ S.E. of these averages. Values greater than one indicate that the modulated grating is more detectable

<table>
<thead>
<tr>
<th>Subject</th>
<th>Mean luminance 5.0 cd/m$^2$</th>
<th>Mean luminance 75 cd/m$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C.F.S.</td>
<td>1.27 ± 0.14</td>
<td>0.75 ± 0.13</td>
</tr>
<tr>
<td>S.K.</td>
<td>1.11 ± 0.23</td>
<td>0.83 ± 0.07</td>
</tr>
</tbody>
</table>

Overall Ratio 0.96 ± 0.08

the simple gratings. Tables 2 and 3 show in fact that the sideband frequencies are not more detectable than the center frequency. For the lower luminance level (Table 2), the central component at 4.0 c/deg was compared within a single run, to a grating of 2.7 c/deg of identical contrast. (The main side-band components are at 2.7 and 5.3 c/deg—see Fig. 2, top.) A 4.0 c/deg was also compared to a 5.0 c/deg grating, where each grating, in this case, was used in separate runs close together in time. The average difference in $d$ for these pairs of gratings and $\pm 1.0$ S.E. of the average are shown in the right-most column of Table 2. A positive value indicates that the 4.0 c/deg grating was the more detectable member of the pair. Table 3 shows similar data for the higher luminance level, where the central component at 4.5 c/deg is compared to a grating of 3.0 or 6.0 c/deg within single runs—see the spectrum in Fig. 2, bottom. The contrast sensitivity at the side-band frequencies is not greater than the sensitivity at the central frequency. Thus, the similar detectability of simple and frequency modulated gratings cannot be accounted for by a more sensitive channel peaked at one of the sideband frequencies.

DISCUSSION

Our basic finding is that an irregular frequency modulated grating and a regular sinusoidal grating of identical contrast are about equally detectable.

The sinusoidal grating however was somewhat more detectable than the FM grating at the higher luminance level (Table 1). This may well be due to the fact that at the higher luminance level, the subjects' contrast sensitivity was less at both sideband frequencies (Table 3) than at the center frequency.

The present results imply that the bandwidth of detection mechanisms near 3–6 c/deg may span one octave, whereas other studies on grating detection, discussed earlier, imply that the bandwidth may be considerably narrower. This apparent conflict will be considered in several steps:

1. First, calculations are presented to show the degree to which different mechanisms are stimulated by FM and quasi AM gratings. And the implications of a peak detection model are considered.

2. The role of probability summation is then examined, and it is concluded that this factor is too small to resolve the conflict over bandwidth.
Table 3. Detectability $d'$ and ±1.0 S.E. for gratings of 4.5 c/deg vs 3.0 and 6.0 c/deg. The average differences in $d'$ for pairs are given in the right column, where positive values indicate that the 4.5 c/deg grating is more detectable. Mean luminance 75 cd/m$^2$.

<table>
<thead>
<tr>
<th>Subject C.F.S.</th>
<th>3.0 c/deg</th>
<th>4.5 c/deg</th>
<th>6.0 c/deg</th>
<th>Average difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.200</td>
<td>0.82 ± 0.38</td>
<td>1.23 ± 0.41</td>
<td>0.44 ± 0.31</td>
<td></td>
</tr>
<tr>
<td>0.222</td>
<td>1.29 ± 0.41</td>
<td>1.49 ± 0.54</td>
<td>0.47 ± 0.29</td>
<td></td>
</tr>
<tr>
<td>0.333</td>
<td>0.37 ± 0.37</td>
<td>1.12 ± 0.37</td>
<td>0.36 ± 0.26</td>
<td></td>
</tr>
<tr>
<td>0.386</td>
<td>2.58 ± 0.65</td>
<td>2.61 ± 0.66</td>
<td>0.47 ± 0.29</td>
<td></td>
</tr>
<tr>
<td>0.200</td>
<td>1.14 ± 0.40</td>
<td>0.66 ± 0.35</td>
<td>0.47 ± 0.29</td>
<td></td>
</tr>
<tr>
<td>0.200</td>
<td>1.20 ± 0.41</td>
<td>0.56 ± 0.32</td>
<td>0.47 ± 0.29</td>
<td></td>
</tr>
<tr>
<td>0.244</td>
<td>0.50 ± 0.36</td>
<td>0.45 ± 0.36</td>
<td>0.47 ± 0.29</td>
<td></td>
</tr>
<tr>
<td>0.366</td>
<td>0.60 ± 0.46</td>
<td>0.60 ± 0.46</td>
<td>0.47 ± 0.29</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Subject S.K.</th>
<th>3.0 c/deg</th>
<th>4.5 c/deg</th>
<th>6.0 c/deg</th>
<th>Average difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.200</td>
<td>0.26 ± 0.27</td>
<td>0.71 ± 0.32</td>
<td>0.49 ± 0.31</td>
<td></td>
</tr>
<tr>
<td>0.244</td>
<td>1.79 ± 0.44</td>
<td>1.75 ± 0.46</td>
<td>0.49 ± 0.31</td>
<td></td>
</tr>
<tr>
<td>0.366</td>
<td>0.65 ± 0.43</td>
<td>1.27 ± 0.48</td>
<td>0.49 ± 0.31</td>
<td></td>
</tr>
<tr>
<td>0.200</td>
<td>0.55 ± 0.45</td>
<td>0.00 ± 0.27</td>
<td>0.49 ± 0.31</td>
<td></td>
</tr>
<tr>
<td>0.222</td>
<td>1.21 ± 0.38</td>
<td>0.92 ± 0.30</td>
<td>0.49 ± 0.31</td>
<td></td>
</tr>
<tr>
<td>0.222</td>
<td>0.79 ± 0.36</td>
<td>0.36 ± 0.34</td>
<td>0.49 ± 0.31</td>
<td></td>
</tr>
<tr>
<td>0.244</td>
<td>1.06 ± 0.37</td>
<td>0.58 ± 0.35</td>
<td>0.49 ± 0.31</td>
<td></td>
</tr>
<tr>
<td>0.244</td>
<td>2.92 ± 0.84</td>
<td>1.09 ± 0.42</td>
<td>0.49 ± 0.31</td>
<td></td>
</tr>
</tbody>
</table>

(3) Finally, possibilities are considered of how the responses of medium-band mechanisms can be linearly pooled to account for the detectability of both FM and quasi AM gratings.

**Stimulation magnitude**

Quantitative estimates of how various gratings stimulate mechanisms of different bandwidth will now be considered. Two bandwidths are examined.

1. A narrow-band mechanism, $N$, given by
   
   \[ \Phi_N(f_p/f) = \exp\left(-\frac{(f/f_p)^2}{\sigma^2}\right) \]  
   
   where $f_p$ indicates peak frequency and $\sigma = 0.17$. The bandwidth is chosen to agree with the data of Sachs et al. (1971) near 14 c/deg and the data of Quick (1973) near 5 c/deg—a full bandwidth of about one quarter octave at half amplitude. Kulikowski and King-Smith (1973) found a narrower channel with $\sigma = 0.11$ near 5 c/deg. However, the value $\sigma = 0.17$ will be used here.

2. A medium-band mechanism, $M$, given by
   
   \[ \Phi_M(f_p/f) = \left[ \frac{e^{-k(f/f_p)^2} - e^{-4k(f/f_p)^2}}{e^{-k} - e^{-4k}} \right]^2 \]  
   
   where $k = 0.462$ is chosen so that the function peaks at $f = f_p$. Blakemore and Campbell (1969) used this expression, which has a one octave full bandwidth at half amplitude, to fit their adaptation data. Further evidence for medium-band channels is provided by studies on apparent contrast following adaptation (Blakemore, Muncey and Ridley, 1973), simultaneous noise masking (Stromeyer and Julesz, 1973), the McCullough effect (Stromeyer, Lange and Ganz, 1973; Stromeyer, Lange and Dawson, in preparation) and facilitation (Stromeyer and Klein, 1974). The one octave bandwidth observed in adaptation studies is apparently not due to physiological non-linearities caused by high-contrast patterns, for the same bandwidth is observed with adapting patterns that are near-threshold (Stecher, Sigel and Lange, 1973). Adaptation studies also show that the channels may be somewhat narrower at higher spatial frequencies than those used in the present study (Blakemore and Campbell, 1969; Stecher et al., 1973). For example, Klein, Stromeyer and Dawson (in preparation) showed, with a signal detection paradigm, that a grating of 60 c/deg produced no threshold change at 10 c/deg (3/4 octave above the adapting frequency), and thus channels near 10 c/deg are not affected by a 6 c/deg grating. (Detection studies with quasi AM gratings also suggest that spatial frequency bandwidths are narrower at higher frequencies—see Introduction.)

The shape of the receptive field (line spread function) of a mechanism is given by the Fourier transform of the spatial frequency function $\Phi(f_p/f)$ of the mechanism

\[ RF_{f_p}(x) = \int_{-\infty}^{\infty} df \Phi(f_p/f) \cos 2\pi[f(x - x_c) - \tau] \]  

Each mechanism can be characterized by its center position $x_c$, peak spatial frequency $f_p$, and phase $\tau$. For an on-center receptive field, $\tau = 0$; for an anti-symmetric mechanism, $\tau = \pm 1/4$, and for an off-center mechanism $\tau = 1/2$. The receptive field of the medium-band mechanisms resembles, for example, the symmetric bar-shaped or antisymmetric edge-shaped receptive fields of simple cells (Stromeyer and Klein, 1974). The receptive field of the narrow-band
The detectability of frequency modulated gratings

<table>
<thead>
<tr>
<th>Spatial frequency $f_p$</th>
<th>Spatial position $x_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.30 0.40 0.50 0.60 0.70 0.80 0.90 1.00 1.10 1.20 1.30 1.40 1.50 1.60</td>
<td>0 1 2 3 4 5 6 7 8 9 10 11 12 13</td>
</tr>
</tbody>
</table>

Fig. 5. A stimulation map which shows the amount of stimulation of mechanisms whose peak spatial frequencies are indicated on the ordinate and whose spatial positions are indicated on the abscissa. The stimulus here is a sinusoidal grating of frequency 100 and contrast 10. On the left side of the figure the mechanisms are assumed to have the narrow bandwidth found by Sachs et al. (1971) and by Quick (1973) as given in equation (6) of our discussion; on the right side, they are assumed to have the medium bandwidth found by Blakemore and Campbell (1969), as given in equation (7). The mechanisms are assumed to be equally sensitive at all spatial positions, and thus stimulation is constant across the grating. (See Discussion.)

The mechanism resembles a grating of regularly spaced, alternating excitatory and inhibitory bands (see Introduction).

The stimulation of a mechanism by a pattern $P(x)$ is determined by the overlap of the receptive field and the pattern (Appendix A). Only mechanisms with the same orientation as the grating will be considered here. Figures 5 to 9 show stimulation of both the narrow-band and medium-band mechanisms by particular sinusoidal, FM, and quasi AM gratings. Each entry in these stimulation maps indicates the amount of stimulation of a mechanism whose peak spatial frequency, $f_p$, is specified on the ordinate and whose center position at a particular place on the grating, $x_i$, is specified on the abscissa. The entry pertains only to the mechanism with the optimal phase, $\pi$ (where $\pi$ can range from 0 to 1). For example, the optimal mechanism centered on a bright bar is an on-center cell ($\pi = 0$) that fires to a bright bar, and the optimal mechanism centered on a dark bar is an off-center cell ($\pi = 1/2$) that fires to a dark bar. The optimal mechanism centered at a zero crossing has an antisymmetric edge-shaped receptive field ($\pi = \pm 1/4$). (Details of the calculations are given in Appendix A.)

The left and right halves of Fig. 5 show, respectively, stimulation of the narrow-band and medium-band mechanisms by a sinusoidal grating at frequency 10 and contrast 10 (units arbitrary).

Figures 6 and 7 show stimulation of the narrow-band and medium-band mechanisms, respectively, by the FM grating used in the present experiment. The grating has contrast 10, as does the sinusoidal grating (Fig. 5), and the three central spectral components fall on the abscissa. The entry pertains only to the mechanism with the optimal phase, $\pi$ (where $\pi$ can range from 0 to 1). For example, the optimal mechanism centered on a bright bar is an on-center cell ($\pi = 0$) that fires to a bright bar, and the optimal mechanism centered on a dark bar is an off-center cell ($\pi = 1/2$) that fires to a dark bar. The optimal mechanism centered at a zero crossing has an antisymmetric edge-shaped receptive field ($\pi = \pm 1/4$). (Details of the calculations are given in Appendix A.)

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The left and right halves of Fig. 5 show, respectively, stimulation of the narrow-band and medium-band mechanisms by a sinusoidal grating at frequency 10 and contrast 10 (units arbitrary).

Figures 6 and 7 show stimulation of the narrow-band and medium-band mechanisms, respectively, by the FM grating used in the present experiment. The grating has contrast 10, as does the sinusoidal grating (Fig. 5), and the three central spectral components fall on the abscissa. The entry pertains only to the mechanism with the optimal phase, $\pi$ (where $\pi$ can range from 0 to 1). For example, the optimal mechanism centered on a bright bar is an on-center cell ($\pi = 0$) that fires to a bright bar, and the optimal mechanism centered on a dark bar is an off-center cell ($\pi = 1/2$) that fires to a dark bar. The optimal mechanism centered at a zero crossing has an antisymmetric edge-shaped receptive field ($\pi = \pm 1/4$). (Details of the calculations are given in Appendix A.)
The stimulus is the same quasi amplitude modulated pattern chosen to represent stimuli that have been shown (1973). The contrast values for all the stimulation maps were used to calculate the peak response. Probability summation provides one possibility for combining the responses in a weighted average.

Assume that a pattern is detected when any mechanism responds too weakly to the FM grating. To resolve this conflict, a different detection model is needed. Next we consider whether probability summation can sufficiently increase the response of narrow-band mechanisms to the FM grating or sufficiently decrease the response of medium-band mechanisms to the quasi AM grating so that a single bandwidth mechanism accounts for all results.

**Probability summation**

Detection may be based on the average of the response of many mechanisms, rather than on simply the peak response. Probability summation provides one possibility for combining the responses in a weighted average.

For an ideal observer, acting according to signal detection theory, probability summation is calculated by the square root of the sum of squares of the detectabilities of independent channels (Green and Swets, 1966). Thus, if two patterns are each detected by different independent channels, then the detectability of the combined pattern is

\[ d' = \sqrt{d'_1^2 + d'_2^2}. \]

To predict detectability, we must know the function that relates detectability \( d' \) to the stimulation magnitude (which, itself, is linearly related to contrast, \( c \)). This function, called the transducer function, has the form

\[ d' = kc' \]

for a \( 90^\circ \) grating and Nachmias and Sansbury (1974) obtained an exponent between 2 and 3. In extensive measurements (Klein et al., in preparation) a single test pattern with three possible contrast values was randomly presented with blank trials during each session. One of the parameters of the minimum chi-square analyses of the signal detection data was the exponent \( t \), where a value \( t = 2.5 \) gave the best fit.

Thus, the response to the superposition of several gratings whose spatial frequencies are sufficiently separated to stimulate different independent channels, is given by

\[ d' = \left( \sum c_i^2 \right)^{1/2} = k \left( \sum c_i^2 \right)^{1/2} \]

where \( c_i \) is the contrast of the \( i \)th component. This formula for combining contrasts does not depend on signal detection theory—the formula is derived in Appen-
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dix B from probability summation as applied to the
psychometric function (obtained with method of con-
stant stimuli).

To predict the detectability of the FM grating by
narrow-band mechanisms, using probability summation,
we assume for simplicity a bandwidth that is nar-
row enough so that each of the three central com-
ponents of the grating stimulate different independent
mechanisms. (The narrow-band mechanisms which
have a 50 per cent fall off 20 per cent away from
the peak are too narrow to produce significant
summation between the central components that are
separated 33 per cent in spatial frequency.) The three
central components have contrast 0.55 times the
contrast of the entire grating. (The other components
contribute little to detection because of the large
exponent t of the transducer function.) Thus,

\[ d_{\text{mod}} = k \left( \sum_{i=1}^{3} c_{i} \right)^{1/2} \]

\[ = k \left[ 3c_{\text{mod}}^{2} \right]^{1/2} \]

where \( c_{\text{mod}} \) refers to the amplitude of the spectral
components of the modulated grating. The ratio of the
predicted detectability of the modulated versus
unmodulated sinusoidal grating can be calculated:

\[ \frac{d_{\text{mod}}}{d_{\text{sin}}} = \frac{k \left[ 3c_{\text{mod}}^{2} \right]^{1/2}}{k c_{\text{sin}}} \]

\[ = \left( \frac{3c_{\text{mod}}}{c_{\text{sin}}} \right)^{2} \]

\[ = \sqrt{3} \left( \frac{0.55}{0.5} \right)^{2} \]

\[ = 04 \pm 01 \]

(9)

where the exponent t of the transducer function is
assumed to be 2.5 \pm 0.5. The observed overall ratio in
the present experiment is 096 \pm 008 (Table 1). Prob-
ability summation is thus not sufficient to make the
FM grating as detectable as the sinusoidal grating,
assuming that the three grating components stimulate
different channels. This argument can be made still
stronger, for the factor \( \sqrt{3} \) in equation (9) may be too
large, since it describes the ideal observer with uncor-
related noise between independent detection channels. A
real observer may not be able to attend equally to all
channels, and thus the noise would be correlated.
Studies on the detection of two superposed sinusoidal
gratings well separated in spatial frequency (Graham
and Nachmias, 1971; Pantle, 1973) suggest that prob-
ability summation is not fully effective. This may be
due to signal uncertainty, which would also reduce
the detectability of the FM gratings below the level
predicted from the present calculations of probability
summation.

Can probability summation, however, predict the
detectability of the quasi AM grating if medium-band
mechanisms are used for detection? This is now con-
sidered. The implicit argument for narrow-band
mechanisms is that medium-band mechanisms re-
spond too strongly to quasi AM gratings according to
the peak detection model. The quasi AM grating with
frequency components 0.8 and 1.0, each with contrast
8, (used to construct Figs. 8 and 9) can be represented
by a trigonometric identity.

\[ P(x) = 8 (\cos 0.8 \ 2 \pi x + \cos 1.0 \ 2 \pi x) \]

\[ = 16 \cos 0.1 \ 2 \pi x \cos 0.9 \ 2 \pi x. \]

(10)

The amplitude of the envelope of this quasi AM pat-
tern, expressed in the arbitrary contrast units, is thus
16(\cos 0.1 \ 2 \pi x), A model based on medium-band
mechanisms, with probability summation, must con-
sider not just the peak contrast of 16, but the contrast
across the entire grating. The table in Appendix B shows
that probability summation yields an effective
contrast of 0.81 of the peak contrast of 16 units, or 130
effective contrast units. This reduction in effective con-
trast is not a sufficient reduction to account for the em-
pirical observation that the quasi AM grating is only
detectable as a sinusoidal grating of 10 contrast
units—the quasi AM grating is thus too detectable
according to probability summation.

Probability summation also can not account for the
observation (Hoeckstra et al., 1974) that the contrast
threshold for sinusoidal gratings of 1 to 7 c/deg (L0
above 25 cd/m²) is about halved when the number of
cycles is increased from two to six. Probability summa-
tion predicts (Appendix B) that tripling the number of
cycles reduces the threshold by only 31/2 = \( \sqrt{3} \) \= 1.3.
A two fold reduction would require a 32-fold increase
in the number of equally activated, independent chan-
nels.

King-Smith and Kulikowski (1974) claim that the
sensitivity of the grating detector is due largely but
not entirely (King-Smith and Kulikowski, 1975) to
probability summation among several line detectors.
However, the sharp, individual lines that they use as
stimuli have low frequency components (which are not
present in gratings) that may aid detectability of the
lines, and thus it is questionable whether the detect-
ability of gratings can be predicted from the detect-
ability of individual lines.

To conclude, probability summation does not suffi-
ciently increase the detectability of FM gratings, using
narrow-band mechanisms for detection. Also, prob-
ability summation does not sufficiently decrease the
detectability of quasi AM gratings, assuming medium-
band mechanisms are used.

Linear pooling

Probability summation predicts that the medium-
band mechanisms will yield too high a detectability for
quasi AM gratings, because the large exponent p used
in probability summation (Appendix B) weights too
heavily the response of mechanisms at the optimal
position. And thus the rather weak responses of
mechanisms at non-optimal positions (at regions of the
grating where contrast is lowest) do not significantly
decrease the response of the total pool of mechanisms.

A pooling mechanism will now be considered which
linearly summates the response of spatially contiguous
medium-band mechanisms. The effective contrast of the
quasi AM grating [represented by equation (10)], for
this pooling mechanism, is 64 per cent of the peak con-
trast, as may be seen in the Table of Appendix B—for
the exponent p = 1. This pooling reduces the effective
contrast to 102 contrast units (64 per cent of the 16
peak units), which implies that the quasi AM grating
should be about as detectable as a sinusoidal grating of
10 contrast units; the two gratings are in fact about
equally detectable. In addition, by limiting the spatial
extent of the pooling to a fixed number of cycles
(depending on luminance), the data of Hoeckstra et al.
(1974) on grating detection as a function of number of
cycles can readily be accounted for.
The linear pooling may be carried out by a mechanism which has inputs from several medium-band mechanisms of similar peak spatial frequencies and different contiguous center positions. The medium-band mechanisms are assumed to respond approximately linearly as a function of contrast, at least within a small range above threshold, and the higher pooling mechanism may linearly summate these responses. (The non-linearity represented by the transducer function, that relates detectability d' to contrast, is thus assumed to occur after the pooling mechanism.) The pooling mechanism has no unique receptive field, for it responds about equally to a bar placed at any position within its large receptive field, and thus a regular grating and a somewhat irregular (FM) grating will produce similar responses. The mechanism does not have a narrow-band frequency response, but rather a response similar to the first-stage medium-band mechanisms. The receptive field of the pooling mechanism is not the Fourier transform of the frequency response since the first-stage mechanisms introduce a non-linearity by acting as rectifiers (due to their slow spontaneous firing rate).

Other schemes for linear pooling can readily be imagined—besides the present physiological model. Linear pooling appears to account for the facts on the detectability of both FM and quasi AM gratings, and narrow-band visual mechanisms may be unnecessary. The present study provides initial evidence that narrow-band visual mechanisms do not exist. Narrow-band mechanisms might serve little purpose since they would poorly encode phase, or position, information, whereas medium-band edge mechanisms could ideally perform this function.

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REFERENCES

APPENDIX A
The stimulation map is calculated from the overlap of the stimulus pattern Ps(x) and the receptive field RFs(x - x1). The receptive field (line spread function) can be written in terms of the mechanism sensitivity curve $\Phi(\omega)$

$$RF_s(x - x_1) = \int_0^\infty df \Phi(\omega) \cos 2\pi f(x_1 - x_1)$$

where $x_1$ is the central position of the mechanism and $\omega$ is the phase of the mechanism. For a bright line mechanism $\omega = 0$; for an antisymmetric edge mechanism $\omega = \pm \frac{1}{2}$; and for a dark line mechanism $\omega = 1/2$.

Since all stimulus patterns used in the present experiment are symmetric periodic functions, they can be expanded in

$$R(s) = \sum_{n=-\infty}^{\infty} R_n$$
The detectability of frequency modulated gratings

The probability that one or more channels detects a pattern is obtained with the method of constant stimuli, in which the observer maintains a constant, low false alarm rate. Probability summation states that the probability that the pattern is not detected by any of several independent channels is

\[ P(c) = \prod_{i} P(c_i) \]

where \( p = 5 \) is gotten by directly fitting the psychometric functions found by Sachs et al. (1971). [The value \( p = 7.8 \) which Quick uses does not fit the data of Sachs et al. (1971) unless an additional correction for guessing is assumed.] The contrast \( c \) is in units of threshold contrast, since \( P(1) = 0.5 \)—the threshold contrast is defined as 50 per cent probability of seeing on the psychometric function. Using this parameterization, the probability that the pattern is not detected by any of several independent channels is

\[ P(c) = \prod_{i} P(c_i) = 2 - 2^c \]

The signal detection theory procedure for combining the output of independent channels gives the same pooling formula since

\[ d' = \sum \left[ c_i^2 \right]^{-1/2} \]

Relating \( d' \) to \( c' \) using the transducer function gives

\[ c'_{\text{eff}} = \left[ \sum c_i^2 \right]^{-1/2} \]

Thus the pooling exponent is related to the transducer exponent by \( p = 2t \). Consider, for example, a pattern of two superposed sinusoidal gratings that are well separated in spatial frequency in order to stimulate different, independent detection channels. If each component is at its own detection threshold, then the effective contrast of the compound grating is \( c'_{\text{eff}} = (1 + 1)^{1/p} = 2^{20} = 1.15 \). Thus by probability summation, the effect of adding the second grating to the first grating, increases detectability by the same amount as would occur if the second grating were not added, but rather the contrast of the first grating was increased by 15 per cent.

As another example, assume that independent detection channels at different spatial positions are stimulated with a grating \( A(x) \cos 2\pi f x \), whose amplitude \( A(x) \) is slowly varying across the grating. The ratio of the effective contrast of this modulated grating to the effective contrast of an unmodulated sinusoidal grating of amplitude \( A_0 \) is

\[ c'_{\text{eff}}(\text{mod}) = \left[ \int A(x)^2 dx \right]^{1/p} \]

with the integral extending over the region of probability summation. For the quasi AM pattern given by equation (10) of the Discussion \( A(x) = \cos 0.1 2\pi x \), the effective contrast of this grating relative to an unmodulated sinusoidal grating of the same peak amplitude and spatial frequency is calculated and shown in the table for different values of the pooling exponent \( p \).

<table>
<thead>
<tr>
<th>Pooling exponent ( p )</th>
<th>Model</th>
<th>Linear summation</th>
<th>Energy (power) summation</th>
<th>Probability summation</th>
<th>Probability summation (Quick)</th>
<th>Peak detection</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_{\text{eff}}(\text{quasi AM})/c_{\text{eff}}(\sin) ):</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.64</td>
<td>0.71</td>
<td>0.81</td>
<td>0.92</td>
<td>1.0</td>
<td></td>
</tr>
</tbody>
</table>
Both the linear summation and the energy summation models are compatible with the premise that quasi AM gratings are detected by medium-band mechanisms (see Discussion).

APPENDIX C

Hoekstra et al. (1974) measured the contrast threshold of sinusoidal gratings as a function of number of bars and luminance. The threshold decreased as the number of cycles was increased up to a saturation point, which depended on luminance but which was independent of spatial frequency within the range tested—1–7 c/deg.

The bandwidth of the detection mechanism can be estimated from the data, assuming a peak detection model in which the threshold is determined by the mechanism that responds maximally. Let it be assumed that the sensitivity $S$ of the mechanism to a pattern $P(x)$ is proportional to the overlap of the pattern and the receptive field $RF$ of the detection mechanism.

$$S_{n,x} \propto \int_{-\pi}^{\pi} RF_{n,x}(x)P(x) dx.$$  

If the frequency selectivity of the mechanism is

$$\Phi(f_p) = \exp\{ - (f_p f_0 - 1)^2 / \sigma^2 \}$$  

where $f_0$ is the peak frequency of the mechanism [same as equation (6) of Discussion], then the receptive field [the Fourier transform of equation (C-2)] is

$$RF_{n,x}(x) = \cos 2\pi f_p x + \alpha E(x).$$  

where the envelope is $E(x) = \exp \{- \pi x f_p \sigma^2 \}$. The sensitivity of the mechanism to a sinusoidal grating of $n$ cycles relative to a very large grating can be expressed

$$S_n / S_0 = \int_{-\pi}^{\pi} E(x) dx \int_{-\pi}^{\pi} E(x) dx$$  

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-x^2/2} dx$$  

where

$$y = (\pi f_p \sigma^2)^{1/2} y$$  

and

$$z = (\pi f_p \sigma^2)^{1/2} n/2.$$  

Thus

$$\sigma = \frac{2 \pi}{n \sqrt{2\pi}} = 0.45 z/n.$$  

For $z = 1.1$ which from equation (C-5) gives $S_n / S_0 = 0.7$, we get

$$2\sigma = 1.0 / n_0.$$  

Thus the full bandwidth, $2\sigma$, is the reciprocal of the number of cycles ($n_0$) needed to obtain 70 per cent of the maximal possible sensitivity value. Inspection of Hoekstra et al.’s (1974) data shows that $n_0$ is 2.3, 3.6, 5.0, 5.5 at mean luminance 2, 25, 165 and 600 cd/m² respectively. Equation (C-8) gives the respective values $\nu = 0.22, 0.14, 0.10, 0.09$. The bandwidths at the three higher luminance levels are compatible with that found by Kulikowski and King-Smith (1973). Hoekstra et al. (1974) calculated bandwidths which were narrower assuming that the detection mechanism responds to a narrow frequency range of the power spectrum of the stimulus.

Experiments with FM gratings throw considerable doubt upon these methods of estimating the frequency selectivity of visual mechanisms.