Gaussian Power with Cylinder Vector Field Representation for Corneal Topography Maps

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Abstract

The corneal maps displayed on commercially available corneal topography instruments are really one-dimensional, defining quantities only along a meridian, and ignoring shape information along any other direction. Both axial and instantaneous power have the drawbacks that there is a singularity at the center data point on the videokeratograph axis, and the computed powers and appearance of the corneal map change depending on the videokeratograph axis or on the location of asymmetries. We propose a new representation for corneal topography maps, *Gaussian power with cylinder vector field*, that has no singularity, and faithfully produces power values and corneal map patterns that are independent of videokeratograph axis or location of asymmetries.

Introduction

In current corneal topography instruments, the power that is displayed represents the curvature only along the radial direction pointing away from the videokeratograph axis. This approach has shortcomings for corneas with irregular shapes such as in keratoconus.

The key idea is that corneal topography should be considered in the context of a general surface. The current corneal maps displayed on all commercially available instruments are really one-dimensional, defining quantities only along a meridian, and ignoring shape information along any other direction. This is like going for a walk over hilly terrain but being forced to walk only along a straight path, wearing blinders, and not being able to look around to see dips and inclines. Even though we think of these cornea maps as contour maps, this is misleading because the parameters being displayed are not true two-dimensional surface quantities, but merely components in the radial direction, measured relative to an assumed center.

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Since both axial and instantaneous power [1][2][3] are measured relative to a videokeratograph axis, these representations of shape have the drawbacks that there is a singularity, which is due to the presence of toricity on the cornea at the videokeratograph axis. The corneal map changes appearance depending on location of the videokeratograph axis.

As we pointed out [8], these displays can lead to misinterpretations of corneal shape when the apex position is decentered, such as diagnosing false positives for keratoconus, overlooking mild keratoconus, and in the case of keratoconus with a decentered cone, underestimating the curvature at the apex of the cone while overestimating the amount of its decentration (these last two points particularly true for axial power). Both axial and instantaneous definitions are inherently one-dimensional, being calculated along a meridional slice.

Our approach and that of [9] abandons this inherently one-dimensional view and instead displays corneal maps based on computing parameters that provide a true two-dimensional measure of surface shape, defined on a general corneal surface, with no assumptions of dependence on videokeratograph axis and with no singularity at the origin. We will illustrate our approach using a keratoconus example. Our two-dimensional representations reduce the problems associated with one-dimensional representations.

Methods

The standard parameter that is displayed on commercially available videokeratographs is *axial power*, and some instruments also offer *instantaneous power*. Discussions of these powers require the concept of the *meridional plane*. The meridional plane contains the videokeratograph axis and the corneal point of interest. Figure 1 shows a meridional plane, depicted as an orange plane. The red dot is the point where the videokeratograph axis intersects the cornea. The corneal point of interest is shown as a green dot.

Axial power is defined as:

$$P_{axial} = (n - 1)/d_{axial} \tag{1}$$

where n is the index of refraction which, by convention, is taken to be the standard keratometric index (SKI) of 1.3375, and d_{axial} is the distance between the corneal point and the axis in the direction of the corneal normal. Axial power is expressed in diopters when the axial distance is given in meters.

Instantaneous power is analogous to the standard mathematical definition of curvature which is simply the reciprocal of the radius of curvature; it is defined as:

$$P_{inst} = (n - 1)/r_{inst} \quad (2)$$

where r_{inst} is the *instantaneous radius*, which is simply the standard mathematical definition of *radius of curvature*. Instantaneous power is expressed in diopters when the radius is given in meters.

From differential geometry [7], it is known that at each point on a smooth surface, there exists a unique direction along which the curvature of the surface is a maximum and another along which it is a minimum (assuming that they are not equal). Furthermore, these two directions will always be orthogonal provided that the surface is continuous in the first and second derivatives at that point; this is a reasonable assumption for the cornea since even in the case of laser ablative refractive surgery, the epithelium would be smooth. The maximum and minimum curvatures are called the *principal curvatures* and the directions along which these principal curvatures lie are called the *principal directions*.

In Figure 2, the yellow cross-sectional plane contains the normal vector at the point of interest (shown as a green dot). Imagine spinning the yellow plane around the normal, and computing the instantaneous curvature in all directions. Three particular directions are of interest. One is the plane that includes the point (shown as a red dot) where the videokeratograph axis intersects the cornea. We will call this plane the *osculating plane*, borrowing a term from differential geometry [6]. The osculating plane is slightly tilted from the meridional plane of Figure 1 if the corneal normal does not lie in the meridional plane. The angle between these two planes is calculated in [10] We call the instantaneous power in the osculating plane the "normal instantaneous power" and the instantaneous power in the meridional plane the "meridional instantaneous power". The other two directions of interest are the the principal directions, as mentioned above. Figure 3 shows the planes corresponding to these two principal curvature directions. The planes corresponding to the minimum and maximum curvature directions are shown in blue and red, respectively.

Similar to the conversion from instantaneous radius to instantaneous power, each of these principal curvatures is expressed in diopters, by multiplication by $n - 1$, where n is the SKI of 1.3375, and the curvature is expressed in m^{-1}.At each point on the surface of a cornea, we can calculate the maximum and minimum power. Their arithmetic mean is the *mean sphere*, their difference is *cylinder*, and their geometric mean (square root of the product) is *Gaussian power*.† For a more detailed presentation, the reader is referred to

† It may be preferable to use the word "curvature" (in dioptric units) rather than "power" since the quantity being expressed is a representation of shape rather than of refractive power. That is the strategy followed in [10][11]. In the present paper, however, we will follow the clinical convention of using "power".

[6]. Since there is a strong mathematical correlation between the arithmetic and geometric means, the mean sphere is very similar to Gaussian power; thus, we will consider only Gaussian power and not mean sphere.

In an earlier article in this journal, we described our algorithm for reconstructing the shape of the cornea, which goes beyond present algorithms in that the rays are not assumed to lie in the meridional plane [12][13][14]. From the corneal shape provided by our algorithm, we can calculate, for all points on the cornea, the principal curvatures, and hence the principal powers, and thus the mean sphere, cylinder, and Gaussian power, all analytically in closed form. Furthermore, since our algorithm constructs a complete mathematical surface representation of the cornea, it is also possible to compute other parameters such as axial and instantaneous power, which can then be used for comparison purposes. Although our algorithm facilitates the calculation of all these parameters, similar computations could be derived for other methods for the reconstruction of corneal shape. For example, these parameters could be calculated numerically based on height data from discrete height maps that are produced by present corneal topography units.

We will first consider simulated data, using a hyperbolic model of keratoconus [9]. To explain our simulated keratoconic model, we first look at a graph of axial power, as a function of distance, as shown in Figure 4. To simulate the cornea, we use a perfect sphere centered at the origin with constant axial power over its surface. This is represented in the graph as a yellow straight line. The power is denoted P_{sphere}, and is labeled in red. Now, to simulate keratoconus, we represent the "cone" on the cornea by removing a section of the sphere and replacing it with a surface of revolution formed from a hyperbola. Considering this hyperbola between $-t$ and t, its axial power is represented as the yellow curve shown in Figure 5. The maximum power of the cone is denoted P_{cone}. The cornea is represented by three free parameters: the power of the sphere (P_{sphere}), the maximum power of the cone (P_{cone}) and the half-width of the cone (t). The eccentricity e of the hyperboloid ($e^2 > 1$) can be expressed in terms of these three parameters as

$$e^2 = \frac{r_{sphere}^2 - r_{cone}^2}{t^2}, \tag{3}$$

where r_{cone} is the apical radius of curvature of the "cone" region and r_{sphere} is the radius of curvature of the sphere, related to the paraxial powers P_{cone} and P_{sphere} by

$$P_{cone} = \frac{n-1}{r_{cone}} \tag{4}$$

and

$$P_{sphere} = \frac{n-1}{r_{sphere}} \tag{5}$$

where n is the index of refraction which again is taken to be the SKI of 1.3375.

To calculate the Gaussian power for a rotated hyperboloid, we first calculate the Gaussian power for the case where the axis of symmetry is aligned with the videokeratograph axis and then rotate that map. We can do this because Gaussian power is invariant under this rotation since it is intrinsic to the surface at each point. Gaussian power is easy to calculate for the axially symmetric case since the principal curvatures are aligned with the sagittal and tangential directions. A proof of this is given in [6]. The powers along the sagittal and tangential directions are the axial and instantaneous powers, respectively. These principal powers are [15]:

$$P_{axial} = \frac{P_{cone}}{(1 + (\frac{ey}{r_{cone}})^2)^{1/2}} \tag{6}$$

$$P_{instantaneous} = \frac{P_{cone}}{(1 + (\frac{ey}{r_{cone}})^2)^{3/2}} \tag{7}$$

where y is the perpendicular distance from the reference axis to the corneal point.

By definition, P_{Gauss} is the geometric mean of the two principle powers; thus, the Gaussian power for an axially symmetric hyperboloid is

$$P_{Gauss} = \frac{P_{cone}}{1 + (\frac{ey}{r_{cone}})^2} . \tag{8}$$

Recall that cylinder is the difference between the maximum and the minimum powers (the principal powers) at each corneal data point. To provide a complete description of the cornea, we need to illustrate the principal directions over the cornea. Thus, we augment the Gaussian power map by overlaying a cylinder vector field, where each vector shows the orientation of the minimum power. Since vectors can represent more than just orientation, we use the length of vectors to depict the amount of cylinder; that is, the length of the vectors are scaled according to the amount of cylinder at each point. We use this scaling for clarification: the larger the difference between the maximum and the minimum powers, the more important is the direction of the vectors. Also, recall that the minimum and maximum powers are orthogonal at any corneal data point (as long as they are nonzero). Although our system allows the display of both the minimum and maximum powers, we propose the use of the minimum power. This shows the least curved ("flattest") direction; for example, this would show the axis of a cylindrically-shaped surface. The Gaussian power map with a cylinder vector field display is shown for several examples in the next section on "Results".

Results

1. Example of Simulated Keratoconic Data

In the following figures, P_{sphere} and t are held constant at 45 diopters and 2 mm, respectively. We vary the remaining parameter of the cone, the central axial power P_{cone}. We also rotate the cone around so that it is not fixed in the center of the cornea. We use spherical coordinates ϕ (latitude) and θ (longitude) to describe the position of the cone on the sphere. We are looking down on the sphere from above the north pole.

Each figure contains the same four images: In the middle is a mockup showing the center of the simulated cone. The size of the cone in this mockup is calibrated to scale. In the upper left is instantaneous† power, in the upper right is axial† power, in the lower left is Gaussian power with cylinder vector field, and in the lower right is a height map, computed as the radial difference (in micrometers) between the surface and a reference sphere. For the simulated keratoconus example in Figure 6, the reference sphere is taken to be the 45 diopter base sphere. This difference map is similar to the fluorescein map used in fitting contact lenses.

Figure 6(i) shows the spherical model with P_{cone} equal to P_{sphere}. In this case, there is no keratoconus, and the corneal shape is simply a 45 diopter sphere. Note that the three visualizations of power are equal and constant, since instantaneous power, axial power and Gaussian power are all constant for a sphere. The difference map shows that there is no difference between the model and reference sphere.

Next, Figure 6(ii) shows the model with P_{cone} set to 57 diopters of axial power, increasing slightly over P_{sphere}. The center of the cone is at θ=215 degrees and ϕ=12 degrees. This models mild keratoconus. The eccentricity of this hyperbola is e=2.3. Note that the directions of the minimum curvature are away from the cone (analogous to the electric field which is away from a point charge).

Figure 6(iii) is similar to the previous figure, but P_{cone} has reached its maximum value of 82 diopters of axial power. Note that even though our model of the cone is rotationally symmetric, the axial power map is not. It is crescent-shaped, whereas the Gaussian power with cylinder vector field map is indeed rotationally symmetric. At the border between the sphere and the hyperboloid surface of revolution (that depicts the "cone"),

† Using the nomenclature in [10], we are displaying "meridional" axial power and "normal" instantaneous power.

the slope is continuous and thus the axial power has a smooth transition as shown in the upper right image. At this border, the curvature has a discontinuity which would appear as an abrupt color change in the plots of instantaneous and Gaussian power. This is not seen in our images because we are using a spline representation for the surface and this does some smoothing.

Figures 6(iv) and 6(v) are just rotations of the cone from Figure 6(iii) towards the center of the cornea. Its center is now at ϕ=6 degrees for Figure 6(iv) and ϕ=0 degrees (directly on the north pole) for Figure 6(v). The size of the cone is the same as in the previous image; only its distance from the center is changed.

The sequence from Figure 6(iii) to 6(v) is perhaps the most important sequence of figures of the simulation. Note that the general shape and values of the Gaussian power with cylinder vector field map remain constant when the cone moves towards the center. However, for axial and instantaneous power, the characteristic shape and power of the cone change dramatically (from a large crescent-shape to a small circular shape). Although this is misleading, most current instruments use this method of visualization, nonetheless.

The model of our simulated cone is rotationally symmetric, and maintains a constant shape when moving across and around the cornea. Our new representation, Gaussian power with cylinder vector field, faithfully represents the symmetry and shape invariance. The difference maps shown on the lower right have many characteristics in common with the Gaussian map. The extent of the cone appears smaller in the difference map than in the other maps because of the color chosen for the display. However, axial and instantaneous power fail to faithfully represent the symmetry and shape invariance of the cone, erroneously showing the cone region as significantly changing shape as it moves around cornea. This is also the case for real data, which we will now demonstrate.

2. Example of Clinical Keratoconic Data

We will now compare four topography representations using corneal data from one patient's eye. Specifically, we will compare the axial and instantaneous power representations, which are the usual approaches of most current systems, to the Gaussian power with cylinder vector field representation, and to a height map, for this corneal data. The height map is computed as the radial difference between the surface and a reference sphere. Consider the data from keratoconic patient BAB who has a "cone" in the lower right region of his cornea (on the lower left from our viewpoint). Figures 7 and 8 show two different views of this cornea. In Figure 7, the patient looked directly into the center of the videokeratograph. This is called *regular fixation*. In Figure 8, the patient shifted his gaze direction up towards his left (our right) so that the cone aligns with the center of the videokeratograph; we call this *conic alignment*.

There are several important characteristics to the regular fixation axial power and instantaneous power corneal maps. First, even though this patient's keratoconic region is very close to rotationally symmetric, these maps are not. Second, the center of the cone is hard to pinpoint in both these maps.. Third, the maximum axial power of the cone is shown as 57 diopters, which we will see is inaccurate.

Let us compare the two axial power maps generated from the different gaze directions in terms of the three characteristics mentioned above. First, the cone shape has changed significantly, and it is now almost symmetric since the cone is in the center of the picture. Second, the center of the cone is easy to identify here, whereas it was difficult to pinpoint in the previous map. Third, the maximum power of the cone in the regular fixation axial power map was 57 diopters. In the conic alignment axial power map, the maximum power is 78 diopters. This is a 21 diopter difference between the axial power of two fixations of the same eye -- almost a 37% difference, as we showed in [2][5]. Thus, axial power yields two conflicting descriptions of the cone due only to changing the direction of the patient's gaze.

Now let us focus on the lower left map in Figures 7 and 8 which show the Gaussian power with cylinder vector field representation corresponding to the same two gaze directions. In the regular fixation map (Figure 7), the cone is depicted as rotationally symmetric, as it should be, the center of the cone is easy to identify, and the maximum power of the cone is 84 diopters.

In the conic alignment map (Figure 8), we see first that the shape remains relatively symmetric, as was the case for the regular fixation Gaussian power map. (The somewhat diamond-shaped appearance is an artifact of the underlying corneal reconstruction algorithm which uses a rectangular-oriented representation; this is separate from the use of Gaussian power.) Second, the center of the cone is still simple to discern, as before. Third, the maximum power of the cone is still 84 diopters. This is an important point; it indicates that Gaussian power is invariant with respect to gaze direction.

Discussion and Conclusions

Current videokeratographs use axial power, and sometimes instantaneous power, corneal maps. In either case, this provides a single unified representation which enables the corneal shape to be reconstructed. The instantaneous and axial power maps contain similar information since the axial power is the average of the instantaneous power from the videokeratograph axis to the corneal point of interest, provided that both are defined in the meridional plane. We defined instantaneous power in the osculating plane so this connection between axial and instantaneous powers is only approximate, but the error is very small [10].

Instead of axial power and instantaneous power, we propose an alternative display using Gaussian power with cylinder vector field representation. Such an approach has considerable advantages.

Both axial and instantaneous power have shortcomings insofar as there is a singularity at the center corneal data point on the videokeratograph axis, and the computed powers and appearance of the corneal map change depending on the fixation axis or on the location of asymmetries.

Instantaneous power shares the independence from fixation axis only in a limited manner since instantaneous power (defined in the osculating plane) is unchanged if the eye is rotated such that the point where the videokeratograph axis intersects the cornea remains in the osculating plane. For any other eye rotation, the instantaneous power is not constant.

However, our new Gaussian power with cylinder vector field representation overcomes the problems of the axial and instantaneous powers, and faithfully produces power values and corneal map patterns that are independent of fixation axis or location of asymmetries.

One potential drawback of Gaussian power is that it would be imaginary if the product of principal powers were negative, although this does not seem to occur for even the most irregularly shaped corneas.

The spherical difference representation (also called a "height" map) shown in the lower right frame of the figures shares most of the advantages of the Gaussian representation. These two maps look similar except that the Gaussian map is a "sharpened" version of the difference map. One problem with the difference map is that the reference sphere is unspecified. Typically, the reference sphere is chosen to minimize the mean square error of the difference map, but this depends on the area chosen for minimizing the error. Some advantages of the difference maps are discussed in [16][17][18][19]. One advantage of the height difference map in comparison with the Gaussian map is that it is straightforward compute a power map from a height map whereas the computation of a height map from a power map would require an algorithm, such as the arc-step method. One disadvantage of the height representation is that it is difficult to determine the cylinder, whereas this is addressed by the vector field in our new representation.

Furthermore, this representation shows significant promise for the accurate identification of keratoconus. The position of the cone is immediately clear and the region of high power corresponds to the "cone" position.

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FIGURE DESCRIPTIONS

Figure 1.

A meridional plane, depicted as an orange plane. This plane contains the corneal point and the videokeratograph axis.

Figure 2.

The yellow cross-sectional plane contains the normal vector at the point of interest (shown as a green dot).

Figure 3.

The planes corresponding to the minimum and maximum curvature directions are shown in blue and red, respectively.

Figure 4.

This graph illustrates the base corneal model without keratoconus. The model is a simple sphere with constant axial power across its surface. This is represented here as a yellow straight line when plotting distance versus axial power. The power is denoted P_{sphere}, and is labeled in red.

Figure 5.

To model keratoconus, a section of the sphere is removed and replaced with a surface of revolution formed from a hyperbola. The axial power associated with the hyperbola between $-t$ and t is shown as the yellow curve. The maximum power of the cone is denoted P_{cone}.

Figure 6(i).

This shows a mockup depicting the center of the simulated cone (middle), instantaneous power (upper left), axial power (upper right), Gaussian power with a cylinder overlay (lower left) and the height map, or radial difference from the reference sphere (lower right). The center of the cone is at ϕ=12 degrees and θ=215 degrees. Here P_{cone}=P_{sphere}=45 diopters; thus, there is no keratoconus and all power maps are constant.

Figure 6(ii).

Same as Figure 6(i) except P_{cone} is set to 57 diopters of axial power, slightly larger than P_{sphere}. The cone is shown in green in the model.

Figure 6(iii).

The parameter P_{cone} has now reached its maximum value of 82 diopters.

Figure 6(iv).

The parameter P_{cone} is fixed at 82 diopters and the cone is rotated towards the center of the cornea. Its center is now at ϕ=6 degrees.

Figure 6(v).

The center of the cone is now at ϕ=0 degrees (directly at the north pole).

Figure 7. The four views of regular fixation (patient looking directly into the center of the videokeratograph): instantaneous power (upper left), axial power (upper right), Gaussian power with a cylinder overlay (lower left) and the radial difference from a reference sphere (lower right).

Figure 8.

The four views of conic alignment (patient shifting his gaze direction up towards his left so that the cone aligns with the center of the videokeratograph): instantaneous power (upper left), axial power (upper right), Gaussian power with a cylinder overlay (lower left) and the radial difference from a reference sphere (lower right).