

INTERPRETING NONLINEAR SYSTEMS:

THE THIRD ORDER KERNEL OF THE EYE MOVEMENT CONTROL SYSTEM

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ABSTRACT

White noise stimuli are useful for analyzing nonlinear systems. A technique has been developed which improves the temporal resolution of the measured response. The role of the third order Wiener kernel is examined in an application to the eye movement control system. Unusual aspects of that system are explored.

0. INTRODUCTION

Although the Wiener technology (1) for nonlinear systems analysis has been available for more than twenty years, its full power has not yet been realized. But researchers are now beginning to learn how to tailor the stimulus to the system being measured. They are learning how to understand the meaning of the higher order kernels, not just treating them as an identification fingerprint. This gradual accretion of knowledge may enable the Wiener techniques to play an important part in attempts to understand complex systems.

This paper is organized into the following sections: 1. Kernels are defined. Higher order kernels are interpreted as producing temporally local adaptation in lower order kernels. 2. A stimulus is introduced (called super-noise) which is able to avoid the trade-off between statistical accuracy and temporal resolution. 3. The cross-correlation technique for measuring the kernels is presented along with formulae on how to relate kernels obtained using different stimuli. 4. A set of units which may aid kernel interpretation is discussed. 5. The eye movement control system is focussed upon largely because it presents several unusual problems for Wiener analysis. The appropriate choice of stimulus is critical. 6. The Volterra kernels of the eye movement control system are introduced. This system is ideally suited for learning about the function of the diagonal elements of the third order kernel. 7. The relevance of crosskernels is discussed. 8. Concluding remarks are made.

1. KERNELS AND LOCAL ADAPTATION

The output (response), $y(t)$, of an analytic, time invariant, stable system can be related to the input (stimulus), $x(t)$, through a Volterra functional expansion:

$$y(t) = \sum_n \sum_{\tau_j} \delta^n g_n(\tau_1 \dots \tau_n) \prod_{i=1}^n x(t-\tau_i) \quad (1)$$

where δ is the sampling time. The interpretability of the higher order kernels (g_n for $n > 1$) may be aided if Eq. 1 is rewritten as:

$$y(t) \triangleq \sum_{\tau_1} \delta g_1(t, \tau_1) x(t-\tau_1) \quad (2)$$

$$g_1(t, \tau_1) \triangleq g_1(\tau_1) + \sum_{\tau_2} \delta g_2(t, \tau_1, \tau_2) x(t-\tau_2) \quad (3)$$

$$g_2(t, \tau_1, \tau_2) \triangleq g_2(\tau_1, \tau_2) + \sum_{\tau_3} \delta g_3(t, \tau_1, \tau_2, \tau_3) x(t-\tau_3) \quad (4)$$

Throughout this paper the contribution from fourth and higher order kernels will be assumed negligible. From Eq. 2, $G_1(t, \tau)$ is the effective response at time t caused by a flash at $t-\tau$. The nonlinearities of the system (Eq. 3), cause the effective impulse response, $G_1(t, \tau)$, to change, depending what other stimuli were present near the time t . According to Eq. 3, rather than looking at the second order kernel in isolation, it may be helpful to consider the second order kernel as causing the rapid (local) adaptation of the first order kernel (Eq. 3). Similarly, Eq. 4 can be interpreted as the third order kernel causing the rapid adaptation of the second order kernel. For example, Larkin (2) showed that the second and third order kernels of the ERG behave in this manner. That is, the ERG is strongly inhibited just after the background luminance is increased due to one or two prior flashes (2nd order kernel), but quite rapidly the inhibition is inhibited (3rd order kernel) if the string of prior flashes is extended.

By using a cleverly designed fluctuating stimulus it is possible to measure the system's kernels with great speed (compared to using double and triple flashes) and with no distortion (compared to using a sum of sinusoids).

2. SUPERRESOLUTION AND TERNARY STIMULI

With conventional white noise stimuli there is a trade-off between accuracy of the higher order kernels (signal-to-noise ratio) and temporal resolution (the sampling time, δ). To illustrate this trade-off, consider the following ternary stimulus. At every succeeding interval, Δ , there is a probability $p/2$ of having a positive flash (or a rightward step in target position), a probability $p/2$ of an equal negative flash (or a leftward step) and a probability $1-p$ that the stimulus remains unchanged. Consider the following two stimuli for measuring the ERG: $x_1(t)$ has $\Delta = 5$ msec, $p = .1$ and $x_2(t)$ has $\Delta = 15$ msec, $p = .3$. In both cases we take the stimulus to be a luminance pulse of 5 msec duration such that during the pulse the luminance goes to either twice the background level ($x = +L$) or to 0 ($x = -L$). Since in both cases the average time between pulses is $\Delta/p = 50$ msec, both stimuli will appear very similar. The kernels obtained will also be similar. The variance of the two first order kernels will be the same since the first order kernel is the response, time-locked onto each flash, and both stimuli have an equal number of flashes. For the second order kernel, however, the signal-to-noise ratio becomes poorer as p gets smaller since the response is time-locked onto pairs of flashes and there are fewer stimulus events contributing to the larger number of interflash intervals.

In the ERG example mentioned above, an experiment would take 3 times as long with $x_1(t)$ than $x_2(t)$ ($p = .1$ compared to $p = .3$) in order to achieve the same accuracy for the second order kernel. On the other hand the temporal resolution gets poorer as p gets larger (5 msec compared to 15 msec).

By modifying the "whiteness" of the stimulus, it is possible to obtain both a good signal-to-noise ratio and good temporal resolution. This technique which we call superresolution is based upon sampling the stimulus and response faster than Δ , where Δ is how often a decision is made on whether to produce a positive or negative flash.

Using $x_2(t)$ as the stimulus ($\Delta = 15$ msec), but using $\delta = 5$ msec as the sampling time allows the kernels $g_1(\tau)$, $g_2(\tau, \tau + n\delta)$, $g_3(\tau, \tau + n_1\delta, \tau + n_2\delta)$ to be measured with 5 msec resolution for τ (time after the last flash), but 15 msec resolution in the time between flashes. A comparison of the autocorrelation functions of the normal noise, $x_1(t)$ and the super-noise, $x_2(t)$ is instructive:

$$\begin{aligned}
 \overline{x_1(t)x_1(t+\tau)} &= \overline{x_2(t)x_2(t+\tau)} && \text{for all } \tau \\
 \overline{x_1^2(t)x_1^2(t+\tau)} &= \overline{x_2^2(t)x_2^2(t+\tau)} && \text{for } \tau \neq 0 \\
 \overline{x_2^2(t)x_2^2(t+\tau)} &= 0 && \text{for } \tau \neq n\delta \\
 \overline{x_2^2(t)x_2^2(t+\tau)} &= \overline{x_2^2(t)} \Delta/\delta && \text{for } \tau = n\delta \neq 0 \\
 \overline{x_1^4} &= \overline{x_2^4}
 \end{aligned}
 \tag{6}$$

The comb-like autocorrelation properties of $x_2^2(t)$ must be kept in mind when the Volterra expansion (Eq. 1) is inverted to obtain the kernels. The next section shows how Eq. 1 can be inverted even when $x(t)$ is super-noise.

Cross-correlating the input and output gives

$$H_1(\tau) \stackrel{\Delta}{=} \overline{y(\tau)x(t-\tau)} = \sum_{\tau_1} \delta g_1(\tau_1) \overline{x(t-\tau_1)x(t-\tau)} + \sum_{\tau_1, \tau_2} \delta^2 g_2(\tau_1, \tau_2) \overline{x(t-\tau_1)x(t-\tau_2)x(t-\tau)} + \sum_{\tau_1, \tau_2, \tau_3} \delta^3 g_3(\tau_1, \tau_2, \tau_3) \overline{x(t-\tau_1)x(t-\tau_2)x(t-\tau_3)x(t-\tau)} \quad (7)$$

In order to simplify Eq. 7 we make the following assumptions: 1. $x(t)$ is symmetric, which means the value $-x$ is as likely to occur as $+x$. This assumption implies $\bar{x} = 0$ and $\overline{xxx} = 0$. 2. The stimulus has a white spectrum, which means $\overline{x(t)x(t-\tau)} \rightarrow 0$ if $\tau \neq 0$. 3. The expectation values just mentioned (\bar{x} , \overline{xxx} , and $\overline{x(t)x(t-\tau)}$) can be guaranteed to vanish by using a full cycle of shift-register noise. Care must be exercised to ensure the appropriate shift sequence is used so that the spurious expectation values of \overline{xxx} are outside the region of interest (3). If a shift-register full cycle is not used then the run must be sufficiently long to make the above expectation values sufficiently small. With these assumptions, Eq. 7 becomes

$$h_1(\tau) = H_1(\tau)/P \quad (\text{where } P = \overline{x^2} \delta \text{ is the power density.})$$

$$= g_1(\tau) + 3 \sum_{\tau'} \delta^3 g_3(\tau, \tau', \tau') \overline{x^2(t-\tau)x^2(t-\tau')}/P$$

$$= g_1(\tau) + 3P \sum_n \Delta g_3(\tau, \tau + n\Delta, \tau + n\Delta) + \delta g_3(\tau\tau\tau) Q_4 \quad (8)$$

where $h_1(\tau)$ is called the first Wiener kernel and Q_4 is given by $Q_4 = (\overline{x^4/x^2} - 3\overline{x^2})\delta^2$ (see Klein and Yasui (4) for a general derivation). In obtaining Eq. 8 the super-noise autocorrelations (Eq. 6) were used.

Two methods are available for determining $g_1(\tau)$ in Eq. 8: 1. Use a very weak stimulus so the power density, P , is very small. This can be accomplished with well separated flashes. But this method produces a very poor signal-to-noise ratio. 2. Subtract off the nonlinear terms which can be estimated by:

$$h_3(\tau_1, \tau_2, \tau_3) = \frac{1}{3!} \overline{y(t)x(t-\tau_1)x(t-\tau_2)x(t-\tau_3)}/P^3 \quad \text{for } \tau_1 \neq \tau_2 \neq \tau_3 \quad (9)$$

Diagonal values of the kernel are needed in Eq. 8 and they can be obtained by interpolating from the off-diagonal values found by Eq. 9. Presumably the stimulus interval Δ is small enough for interpolation to be satisfactory. For any stimulus other than a binary stimulus $h_3(\tau_1, \tau_2, \tau_3)$ can be calculated directly by crosscorrelation (4,5). If $x(t)$ is a ternary stimulus then $h_3(\tau, \tau, \tau)$ cannot be calculated directly.

From Eq. 8 it is straightforward to express the Wiener kernels measured with stimulus x_a in terms of those measured with x_b . We assume both x_a and x_b are symmetric for simplicity. See Klein and Yasui (4) for the nonsymmetric case.

$$h_1^a(\tau) = h_1^b(\tau) + 3(P^a - P^b) \sum_{\tau'} \delta h_3^b(\tau, \tau', \tau') + (Q_4^a - Q_4^b) h_3^b(\tau, \tau, \tau) \delta \quad \dots \quad (10)$$

If $x(t)$ is Gaussian then the dependence of the kernels upon power density (P) and mean luminance (\bar{x}) takes on the elegant form:

$$\frac{dh_n(\tau_1 \dots \tau_n)}{dP} = \frac{(n+2)(n+1)}{2} \sum_{\tau} \delta h_{n+2}(\tau_1 \dots \tau_n, \tau, \tau) \quad (11)$$

$$\frac{dh_n(\tau_1 \dots \tau_n)}{d\bar{x}} = (n+1) \sum_{\tau} \delta h_{n+1}(\tau_1 \dots \tau_n, \tau) \quad (12)$$

Finite changes in P or \bar{x} can be expressed as a Taylor's expansion using iterations of Eqs. 11 or 12 respectively, to obtain the expansion coefficient.

4. APPROPRIATE UNITS

The meaningfulness of the higher order kernels (and even the first order kernel) is greatly enhanced if careful attention is paid to the units by which $x(t)$ and δ are measured. There are two general cases to consider. First consider the case where $x(t)$ and $y(t)$ are in different units (e.g., luminance and voltage for the ERG). In that case the kernels should have the same units as the response, and $x(t)$ and δ should be measured in units which are

natural for the system. A system which adapts (like the ERG) tends to respond to percent change rather than absolute change in luminance. Thus $x(t)$ could be measured in units of the average luminance, L , in which case $x = -1, 0, 1$ would correspond to luminances, $0, L$ and $2L$, respectively. The unit of time can be chosen so that the size of the power density, $P = \overline{x^2}$, is meaningful. The relative contribution of the nonlinear terms is controlled by P , so it is sensible to have $P \approx 1$ at the point when the nonlinear effects become appreciable. For example, the time unit, T_M , could be that average interflash interval at which the second order kernel's contribution to the response becomes about as large as the first order kernel's contribution. For the ERG, T_M depends on the background luminance and varies from $T_M > 100$ msec at scotopic levels to $T_M < 40$ msec at photopic levels. With these definitions the first and second order kernels should be about the same size. Relationships such as

$$\overline{y^2} = P \sum_{\tau} \delta h_1^2(\tau) + P^2 \sum_{n, \tau} \Delta \delta h_2^2(\tau, \tau+n\Delta) + \dots \quad (13)$$

and

$$h_1(\tau) = g_1(\tau) + P \sum_n \Delta g_3(\tau, \tau+n\Delta, \tau+n\Delta) + \dots \quad (14)$$

can be meaningfully used if the units are sensible.

The second general type of system is one in which $x(t)$ and $y(t)$ are in the same units. For example, they might be angular velocity of target and eye. Or they might be input and output voltage of some black box. Unfortunately if x is not unitless, then the higher order kernels will all have different units making direct interpretations difficult. A solution is to use one set of units for the $x(t)$ in Eq. 2 and different units for the $x(t)$ which appears in Eqs. 3-5. If the $x(t)$ which appears in Eq. 2 has the same units as $y(t)$ then $G_1(t, \tau_1)$ becomes the unitless gain of the system. For the eye movement system a reasonable time unit for δ in Eq. 2 would be the duration of a saccade which might be ~ 15 msec for the small saccades to be considered here. In these units $G_1(t, \tau)$ is the probability of a saccade at time t due to a stimulus at time $t-\tau$. If the higher order kernels in Eqs. 3-5 are to have the same units as $G_1(t, \tau)$ then in those equations $x(t)\delta$ must be unitless. We shall take $x(t)\delta = \pm 1$ for a clockwise or counterclockwise jump in these equations. In this fashion we have incorporated the linear dependence of saccade sizes. As discussed in the next two sections the nonlinearities of concern to us are those involved with the timing of the eye movement control system.

5. A STIMULUS FOR PROBING THE EYE MOVEMENT CONTROL SYSTEM

Previous attempts to stimulate the eye movement system with a rapidly moving spot have been stymied because the eye quits tracking at about 2 Hz (6,7). This is unfortunate because many interesting features of the eye movement control system require a temporal resolution of better than 20 msec. (Cognitive psychology has learned a lot from 10 msec resolution of reaction times.) Two new ideas have made it feasible to dramatically increase the measurable bandwidth: 1. The superresolution technique discussed earlier. 2. The target position should follow a Brownian motion rather than a random motion. Brownian position means the velocity is random. The eye can't track random positions because of the 200 msec refractory period between saccades. By the time the eye saccades to where the target was, the target has probably moved to where the eye was. The eye soon realizes that it can follow the target better by not moving. With Brownian motion it always pays to catch up to where the target was because that position is a good predictor of where the target will be.

As an example of a stimulus which may be good for probing the saccadic system, consider 32 little lights spaced one degree apart around a circle of about 5 degrees radius. At any one instant only one light is on. Every $\Delta = 50$ msec there is a probability p that the light will turn off and that one of the two adjacent lights (randomly chosen with 50% probability for each) will turn on.

The stimulus and the response can be sampled as fast as desired, only limited by the speed of the attending computer. (As an example of speed, a \$700 PET microcomputer can easily handle the stimulus generation, and simultaneous on-line computation and display of 1st, 2nd, and 3rd order kernels of the ERG where $\Delta = 15$ msec and $\delta = 5$ msec). The kernels can be smoothed, if desired, to a coarser sampling time after they have been calculated at a finer resolution. The temporal resolution possible with the super-noise Brownian motion is much greater than the resolution with other forms of Brownian motion (such as that produced by a carefully chosen sum of sinusoids).

6: VOLTERRA KERNELS OF EYE MOVEMENT CONTROL

As discussed in Section 4, the first order kernel $h_1(\tau)$ gives the probability of a saccade at a time t after a velocity pulse. For simplicity we'll assume that $h_1(t)$ has a Gaussian shape, with a full bandwidth of 30 msec and a height such that there is unity area under the curve. The response is delayed by 200 msec as shown by the top line on Figure 1. In this

When the pulse probability is taken to be very small so the Wiener and Volterra kernels are similar.

A general feature of the eye movement stimulus defined in the previous section is that all even-order kernels should vanish. Since the response to a counterclockwise stimulus should be symmetrically opposite to a clockwise stimulus, it can be expected that crosscorrelations of the response with even moments of the stimulus will vanish by symmetry. (The even-order kernels will vanish for any symmetric system--such as a forced pendulum--which is stimulated by a symmetric input.)

In the absence of second order kernels, the response (Eqs. 2-4) can be written in terms of Wiener kernels:

$$y(t) \stackrel{\Delta}{=} \sum_{\tau_1} \delta K_1(t, \tau_1) x(t - \tau_1) \quad (15)$$

$$H_1(t, \tau_1) \stackrel{\Delta}{=} h_1(\tau_1) + 3 \sum_{\tau_2 \neq \tau_1} \delta^2 h_3(\tau_1, \tau_2, \tau_2) (x^2(t - \tau_2) - x^2) \quad (16)$$

The totally off-diagonal contribution from the third order kernel ($\tau_2 \neq \tau_1$) has been dropped as have the contributions from fifth and higher order kernels because we shall assume in this section that the power density is very small. This means the likelihood of a pulse is small and therefore the likelihood of three closely spaced pulses is negligible. The full diagonal term ($\tau_1 = \tau_2$) is excluded from the summation in Eq. 16 as a consequence of the symmetric ternary nature of the stimulus together with Eq. 8.

Information about the third order kernel comes from experiments which use a double pulse of velocity (8). The best data for our purpose was obtained by Komoda, et al. (9) and by Hou (10). Two equal pulses means the target makes two jumps in the same direction. Two opposite pulses causes the target to jump back to the starting point. Suppose the two pulses occur at $t = 0$ and $t = \tau - \tau'$. Using the units discussed in Section 4 ($x(t)\delta = \pm 1$ for a pulse) allows the "effective" impulse response (Eq. 16) to be written as:

$$H_1(\tau, \tau) = h_1(\tau) + 3h_3(\tau, \tau', \tau') \quad (17)$$

since the term containing $P = x^2\delta$ is negligible. The impulse response $h_1(\tau_1)$ gets modified by the second stimulus. It is important to notice that the third order contribution always contributes with a positive sign since $x^2\delta^2 = +1$ in Eq. 16 for both a positive pulse and a negative pulse.

Two possible outcomes of the double step experiment are illustrated in Figure 1. The vertical axis is the probability of response to a test stimulus at $t = 0$ when there is a conditioning stimulus present before (rows a-e) or after (rows f-i) the test pulse. The conditioning pulse is indicated by a line extending above and below the horizontal axis (to indicate the irrelevance of the conditioning pulse's direction).

In rows a and b the interstimulus interval, $\tau - \tau'$ is greater than or equal to 200 msec and the test pulse is not affected by the conditioning pulse, so $h_3(\tau, \tau', \tau') = 0$ for $\tau - \tau' > 200$ msec. In row c the response is delayed because the saccadic response to the conditioning stimulus should occur at about $t = 50$ msec. The intersaccadic refractory period must then delay the response to the test stimulus. In rows d and h instead of always finding one saccade for each stimulus, we assume (for 100 msec between pulses) that there is a 50% chance of reprogramming the responses into a single saccade. There are two ways of reprogramming the saccades. In the left half of Figure 1 is shown the possibility that the first saccade is cancelled and the single saccade occurs 200 msec after the second stimulus. In the right half of Figure 1 is shown the possibility that the second saccade is cancelled and is retroactively absorbed into the first saccade. This is possible since the length of an already planned saccade can be changed in a shorter time than it takes to initiate a saccade. In rows e and g (for 50 msec between pulses) we assume that 100% of the time a single saccade is made, with the possibilities of cancellation or absorption, as discussed above.

If we allow the probability of pulses to increase, the power density will rise and the first order Wiener kernel will begin to show the effects of the third order kernel (see Eq. 10). The two models illustrated in Figure 1 demonstrate several possible effects which can be caused by the third order kernel. It becomes clear how activity after the test pulse can affect the response to the test pulse (see lines g and h of the cancellation model). Also, it should not be surprising if in an actual experiment, the first order Wiener kernel starts responding at 100 msec after the test pulse (see line d of the absorption model).

AN ASYMMETRIC TERNARY STIMULUS AND CROSS-KERNELS

In the double pulse experiments of Komoda, et al. (9), pulses of different sizes were used. Their results have a neat summary in terms of Figure 1: a. If the final position of the target was in the opposite direction from the direction of the first pulse, then the saccade could not be absorbed. The response looked like the left half of Figure 1. b. Otherwise the second saccade could be absorbed into the first saccade (with only a slight extra delay) and the response looked like the right half of Figure 1.

Inspired by the interesting results of Komoda, et al. (9), we now discuss a more complex stimulus for testing the eye movement system. As before, every $\Delta = 50$ msec there is a probability p that the light will jump. But now, if it does jump it will jump clockwise one space 2/3 of the time or counterclockwise two spaces 1/3 of the time. For this stimulus $\bar{x} = 0$, but $\overline{xxx} \neq 0$. It would be possible to obtain all the kernels (including even-order) but they would be difficult to interpret. A better way to analyze the results of such an asymmetric stimulus is to consider the clockwise pulses as belonging to one binary stimulus, $x_+(t)$, and the counterclockwise pulses as belonging to another binary stimulus, $x_-(t)$.

The problems associated with correlated asymmetric binary stimuli and the solutions to these problems will not be discussed here. For this paper we merely point out the rich information contained in the first and second kernels obtained from the two input stimuli. The first order kernels $h_+(\tau)$ and $h_-(\tau)$ should be identical (with the units convention discussed in Section 4), if the probability of a saccade were independent of the size of saccade. The difference between $h_+(t)$ and $h_-(t)$ would for example show whether bigger stimulus jumps are easier to detect and therefore get a quicker response (9). The second order self-kernels $h_{++}(\tau_1\tau_2)$ and $h_{--}(\tau_1\tau_2)$ measure the response to two equal steps in the same direction. The second order cross-kernel, $h_{+-}(\tau_1\tau_2) \sim \overline{y\dot{x}_+x_-}$ measures the response to two steps in opposite directions, where for $\tau_1 < \tau_2$ the second step is half the size of the first (so the final position is in the same direction as the first step), and for $\tau_1 > \tau_2$, the second step is twice the size and opposite in direction to the first step (so the final position is in the opposite direction to the first step). Thus the second order kernels encompass all the interesting cases explored by Komoda, et al. (9).

Preliminary experiments with this type of continuous stimulus have been done on the saccadic system by Ray Hou (10) and on the slow pursuit system by Rick Williams (11) under the guidance of Derek Fender. For the slow pursuit system acceleration pulses replace velocity pulses so $x(t)$ and $y(t)$ are accelerations, and the stimulus is a ring of very closely spaced dots so position cues are eliminated. The results of these experiments confirm the usefulness of the above methods as long as the average time between pulses is greater than 200 msec.

As the pulses get closer together it is no longer possible to ignore the presence of triple pulse interactions which have been ignored so far. Also the dominant role of the refractory period is not adequately treated by the Volterra expansion (1). These problems can be dealt with by inventing a new formalism which involves correlations like $y(t)y(t-\tau_1)x(t-\tau_2)x(t-\tau_3)$. We find by time-locking onto the previous saccade as well as previous stimuli, the main source of uncertainty in the kernel estimation is eliminated. This formalism can also handle situations where the stimulus position depends upon the eye position (6). Luckily, such expansions are outside the Wiener domain so they need not be considered in this paper.

9. CONCLUSIONS

St. Cyr and Fender (12) have made a valiant attempt to understand the nonlinearities of the oculomotor system. Their stimulus was the sum of several sinusoids. But the gains and phase shifts of the sinusoidal components and their interactions are hard to interpret. Furthermore the oculomotor system stopped tracking at about 2 Hz.

On the other hand, double-pulse experiments (8-10) which were discussed in the preceding section have straightforward interpretations in terms of underlying mechanisms. And the pulse technique has excellent temporal resolution. Such experiments place strong constraints on multistage control models (13,14). However, the double-pulse stimuli are not ideal because they are quite dissimilar from real world tracking. One of the main motivations for studying tracking is to learn how it is done in the natural environment. An understanding of pairs of well spaced target jumps is a beginning of knowledge, but far from the end result.

The Wiener kernel techniques discussed in this paper may be helpful in furthering our knowledge about the rapid dynamics of human control systems. If the average time between velocity pulses is around 300 msec the Brownian motion will appear very similar to the sum-of-sinusoid stimuli previously considered. Yet the analysis will have the time resolution and time domain interpretations of the double flash experiment.

A control system with a large refractory period is an unlikely candidate for the Wiener technology. Yet some of the items discussed in this paper may provide useful tools for studying the oculomotor system. The converse is also true: the oculomotor system has been a useful tool for studying the Wiener methodology.

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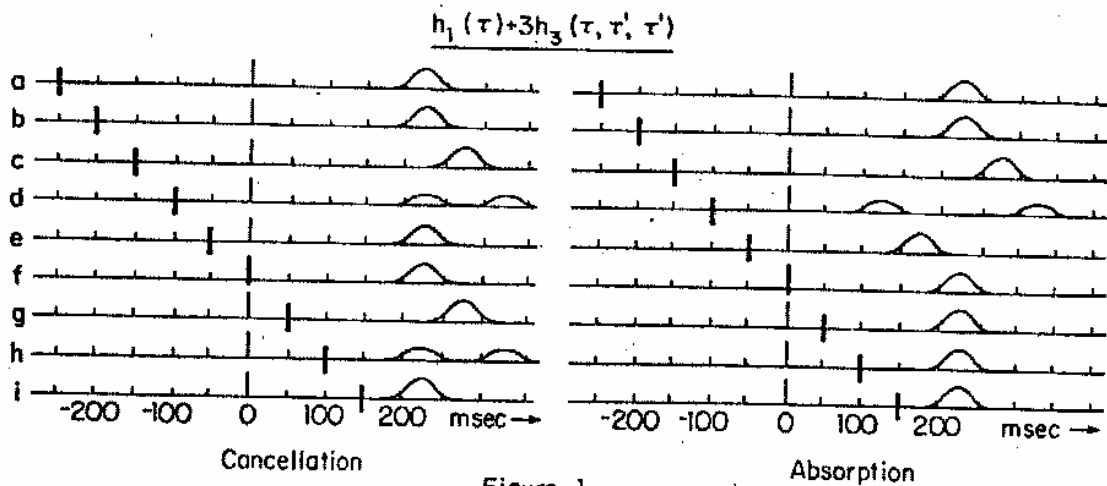


Figure 1