Multizone Bifocal Contact Lens Design

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Abstract

A satisfactory bifocal contact lens has not yet reached the marketplace because of deficiencies in the present designs. The two-zone concentric design allows both the near and far images to be present simultaneously, however the image quality is not very good. Recently several new bifocal contact lenses with multiple zones have been suggested. This paper presents computer calculations of the image quality of several of these new designs. The zone-plate design, with an intermediate blaze angle, gives the best image quality of the lenses that were examined.

Introduction

Persons over 45 years of age lose their ability to accommodate to near objects. Bifocal spectacles provide a solution, whereby one portion of the lens has a dioptric power appropriate to focus on distant objects and a different portion appropriate to focus on near objects. There are many people, however, who refuse to wear glasses and insist on wearing contact lenses. Those using contact lens wearers are the driving force behind the quest for a satisfactory bifocal contact lens.

The most commonly used bifocal contact lens is moulded after bifocal spectacles. Most of the area of the bifocal has a dioptric power appropriate for viewing distant objects, and a small segment, placed off-center, has a dioptric power appropriate for reading. The positioning of the contact lens can be controlled by lowering the eyes so that the lens hits the bottom eyelid. This maneuver places the near-zone of the contact lens in front of the pupil if the lens has the proper orientation. This bifocal has the advantage that it is in view at one time so that when the far-zone is in front of the pupil there is no interference from an out-of-focus near zone and vice-versa. The disadvantage is that most users have difficulty adjusting to the delicate positioning of the eyelids and orienting of the lens required for good vision. This moving bifocal will not be considered further.

Two types of bifocal contact lenses will be considered in this paper. The simplest has two zones arranged annularly (concentrically), with both zones always in view. The advantage of this lens is that it does not require careful positioning. Its disadvantage is that, as will be shown, the image quality is not very good and the image is sensitive to pupil size.

A second type of bifocal contact lens is the zone-plate lens proposed by Allen Cohen. One scheme has multiple zones with the even and odd zones alternating in dioptric power. In Cohen's proposal, the zone radii are chosen so that the alternating zones become a zone-plate. This scheme ensures that a good balance between the even and odd zones is always maintained, independent of pupil size. By combining the focussing power within each zone (geometric optics), with the zone-plate focussing power between zones (physical optics) the image quality is enhanced. In the regime to be explored, diffraction effects are strong and the final image often behaves in counterintuitive ways. Another zone-plate design, advocated by M. Freeman (personal communication) does not have alternating zones and will be shown to have fairly good image quality.

A prime goal of this paper is to present computer calculations comparing the image quality of several zone-plate lenses to the image quality of the two-zone lens and to some non-bifocal lenses. One of our motivations for submitting this paper to SPIE was to elicit comments from experts in this field. This work represents our first venture into diffraction optics. We jumped into doing these calculations without being fully aware of optimal calculational methods and we would appreciate suggestions and comments on our approach.

The schematic eye used for simulations

A single element simplified schematic eye was used for all the computer calculations. The refracting surface is a 90 diopter cornea with indices of refraction of \( n = 1.4 \) outside the eye and \( n = 1.5 \) inside the eye. The distance between cornea and retina is \( 30 \) mm. With this choice of parameters an object at infinity will be in focus on the retina. The different types of contact lenses are obtained by slightly altering the front surface of the cornea (see Table 1). The pupil is located directly on the cornea.

The effect of spherical aberration was examined by using the following formula for the front surface of the cornea:

\[
D = y^2 - 2Rx + x^2
\]

where \( x \) and \( y \) are the locations of a point on the cornea, using a cylindrical coordinate system with an origin at the point where the lens axis (the \( x \)-axis) intersects the cornea, \( R \) is a constant that determines the curvature of the cornea and \( s \) is a constant that controls the amount of spherical aberration. If \( s = 1 \) the
cornea is a section of a sphere with radius R. For our schematic eye the radius, R, is 10 mm as determined by the formula R = (n-1)/D for D = 50 diopters. If s = 1 - 1/n², where n is the index of refraction inside the eye, then the dominant spherical aberration vanishes. In terms of the transformed variables x' = xs and y' = y s² Eq. 1 can be written as the equation for a circle:

\[ x'^2 - 2Rx' + x'^2 \]

so the role of the aberration parameter is to stretch the x axis by an amount greater than the y axis. For the case with vanishing spherical aberration, the cornea has the shape of a prolate ellipsoid (like a football) with a length that is s² = (1-1/n²)² = .745 times the width (for n = 1.5).

The effect of the wavelength of light was examined by calculating with both monochromatic and multicomponent light. The monochromatic light had a wavelength of .56 microns. The multicomponent light had 5 components whose spacing and intensity were chosen to approximate the photopic sensitivity curve. The five components had wavelengths of .48, .52, .56, .60, and .64 microns with a Gaussian intensity envelope of .85 .8409, 1.0, .8409, and .5 respectively. The five point-spread functions were added together to give the total luminance point-spread function.

Six types of contact lenses will be investigated, as shown in Table 1:

<table>
<thead>
<tr>
<th>Name</th>
<th>Number of zones</th>
<th>Diopters odd</th>
<th>Diopters even</th>
<th>Zone radius (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>One-zone</td>
<td>1</td>
<td>50</td>
<td>50</td>
<td>1.697</td>
</tr>
<tr>
<td>Two-zone annular</td>
<td>2</td>
<td>50</td>
<td>53</td>
<td>1.2 1.697</td>
</tr>
<tr>
<td>Fresnel</td>
<td>8</td>
<td>50</td>
<td>50</td>
<td>1.697</td>
</tr>
<tr>
<td>Zone-plate lenses</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Alternating (phase = 0)</td>
<td>8</td>
<td>50</td>
<td>53</td>
<td>.6 1.697</td>
</tr>
<tr>
<td>Alternating (phase = ½)</td>
<td>8</td>
<td>50</td>
<td>53</td>
<td>.6 1.697</td>
</tr>
<tr>
<td>Non-alternating</td>
<td>8</td>
<td>(51.5)</td>
<td>(51.5)</td>
<td>.6 1.697</td>
</tr>
</tbody>
</table>

Table 1

The outside radius of 1.2* 2½ = 1.697 mm was chosen so that an equal amount of light passes through the even and odd zones for the two-zone case. The radius of the kth zone for the 8-zone lenses is 1.697. The square root dependency is used in order to have zones with equal areas. The two-zone annular lens has a central zone for viewing surrounded by an annular zone for near viewing (a 3 diopter add). Three zone-plate lenses will be studied. The first has alternating corneal zones of 50 and 53 diopters that are smoothly joined. The second has the same dioptric powers, but a ½ wavelength phase shift has been added to the light passing through the even zones.

The third zone-plate lens has 8 nonalternating zones and its bifocal nature is caused by using an intermediate blaze angle (to be described). The nature of the three zone-plates is clarified in the discussion following Table 2. The Fresnel lens mentioned in Table 1 is not bifocal. It is a 50 diopter cornea, subdivided into 8 zones, each of which is given a random phase shift. In units of optical wavelengths, the 8 random phase shifts that were used for our calculations are .36, .77, .92, .65, .82, .18, .55, .38. The Fresnel lens is included both to clarify how the zone-plate lenses differ from the Fresnel lens and to clarify why the Fresnel lens has poor optical quality.

Calculation methods

The formula used for calculating the amplitude, A(r), of the light hitting a point, r, on the retina is the sum over all paths \( \sum \exp(i\phi) \).

\[ A(r) = \frac{\sum \exp(i\phi)}{\sum(1)} \]

(3)

The phase, \( \phi \), is proportional to the optical path length, L, by the relation \( \phi = 2\pi L/w \) where \( w \) is the wavelength of the light. The denominator is a normalization factor obtained by setting the phase shifts for each path equal to zero. The number of paths used in the summation was determined as follows: 1. Each zone of the lens was broken up into NSTEP radial subdivisions and 2*NSTEP tangential divisions. 2. The magnitude of A(r) was calculated. For an 8-zone lens with NSTEP = 25 the summation was therefore over 8*25*50 = 10,000 points on the lens. 3. NSTEP was doubled and steps 1 and 2 were repeated until either the magnitude of A(r) changed by less than .00001 or NSTEP become larger than 100 (thus 160,000 was the maximum number of integration points). 4. The final value of A(r) was taken to be 1.1 times the final iteration minus .1 times the next-to-last iteration. The purpose of this final subtraction is to obtain an approximation to Simpson's rule for two-dimensional integrations.

Values of A(r) were calculated for 300 values of r ranging from 0 to 13 min of arc. Thus a point spread function with 5 wavelengths involved calculating about 5*300*160,000 = 2.4*10^8 rays. Each such curve took about 8 hours of execution time on the VAX. Most of our calculations were restricted to monochromatic light in order to gain a 5-fold increase in speed.

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The point spread function, \( P(r) \), that gives the intensity of light falling on each retinal point is equal to the magnitude of \( A(r) \) squared:

\[
P(r) = |A(r)|^2
\]

where \( A^*(r) \) is the complex conjugate of \( A(r) \). The modulation transfer function, \( MTF(f) \), to be plotted, equals the Fourier transform of the point spread function. Since the point spread function for an annular (concentric) lens has radial symmetry, the Fourier transform can be written in terms of a Bessel function:

\[
MTF(f) = \int \frac{\cos(nx)}{\pi} \, dx \, dy \,
= \frac{\cos(nfr)}{\pi} \, r \, dr \, d\theta \,
= \frac{\cos(nfr)}{\pi} \, J_0(nfr) \, r \, dr / \int P(r) \, r \, dr
\]

where the Bessel function is defined by:

\[
J_0(z) = \int \cos(z \cos(\theta)) \, d\theta.
\]

The index of refraction, \( n \), is present in Eq. 5 in order to convert from an angle, \( r \), in image space to a spatial frequency, \( f \), in object space. According to Snell's Law an object space angle is \( n \) times an image space angle.

The calculations were done on a VAXstation using FORTRAN with single precision (4 byte) arithmetic. The following trick was used in order to avoid loss of accuracy due to the loss of significant figures when a number is subtracted from a number that is almost equal. This type of subtraction often occurs in wave optics. For example, the extra path length travelled by a parallel ray before striking the cornea is calculated by solving Eq. 1 for \( x \):

\[
x = \frac{(R - (R^2 - y^2)^{1/2})}{s}.
\]

Since \( R \geq x \) (\( R \) is 10 mm for our schematic eye and \( x \) is less than 0.1 mm) there will be a loss of accuracy due to the subtraction. In particular, for \( y = 0.5 \) mm (this value is the inner radius of the multizonal lenses to be investigated) and \( s = 1 \), the value of \( x \) as calculated by the VAX using Eq. 7 is 18.01586 microns. The correct answer, however, is 18.01623 as calculated by a VAX-20 (which used 5 byte arithmetic). This discrepancy of 0.00037 microns represents about 0.1% of the wavelength of light. Since the summation in Eq. 3 adds up to 160,000 rays for each of the 300 retinal points, an error of 0.1% per ray can accumulate and become significant. This problem was avoided by using an iterative solution to Eq. 1 and using the initial value of \( x \) from Eq. 7:

\[
x = \frac{(y^2 + x^2)^{1/2}}{2R}.
\]

This expression for \( x \) converges rapidly because \( x/R \) is small. When the value of 18.01586 is plugged into Eq. 8, the correct result of 18.01623 microns is obtained. The use of this type of iterative procedure at several points in the computation allowed us to avoid time-consuming double precision arithmetic.

Introduction to blazed zone-plates - the diffraction grating

In order to clarify our method for producing the circular zone-plate bifocal lens, the simpler case of a linear diffraction grating, shown in Fig. 1, is discussed first. In Fig. 1a the width of each slit (zone) is \( d \) and \( h \) is the height of each discontinuity. There will be constructive interference at angles of \( m \) (wavelength) radians, where \( m \) is a positive or negative integer representing the order of interference and \( n \) is the wavelength of the monochromatic light. By changing the height of \( h \) (the blaze height), the amount of light in each interference order can be controlled. For \( n = 0 \), the diffraction grating is identical to a transparent sheet of glass and all the light goes through undeviated into the \( m = 0 \) peak.

If the phase shift across each discontinuity is one wavelength, then the discontinuities could be removed and the diffraction grating replaced by a prism with the same apex angle as each of the facets. In this case all of the light energy goes into the first order maximum (\( m = 1 \)).
discontinuity, \( h \), that produces a one wavelength phase shift can be calculated in terms of the index of refraction of glass, \( n \), by setting the difference of path length in glass (nh) and air (h) equal to one wavelength:

\[
(n-1)\ h = w
\]

Discontinuities other than a full wavelength can be written as:

\[
h = b w / (n-1)
\]

where \( b \) will be called the blaze coefficient. For \( b = 0 \) (corresponding to zero phase shift between zones) all of the light goes into the \( m = 0 \) maximum. For \( b = 1 \) (corresponding to a full wavelength shift between zones) all of the light goes into the \( m = 1 \) order maximum. This paper calculates the image quality of several zone-plate lenses that have an effective blaze of \( b = 0.5 \). The intensity of light into each interference order of a blazed diffraction grating will now be calculated.

The exact amplitude, \( A_m \), of light going into the \( m \)th interference order is calculated by adding up the contributions across each of the zones as specified by Eq. 3. For the nonalternating zone-plate shown in Fig. la, the amplitude is:

\[
A_m = d^{-1} \int_0^d \exp\left(2\pi i (m-b)x/d\right) \ dx
\]

where the factor 1/d corresponds to the denominator of Eq. 3. This integral gives an intensity of:

\[
\text{Intensity} = \left(\sin(\pi b)/(\pi(m-b))\right)^2
\]

For the alternating zone-plates (Figs. lb and lc) the integral (Eq. 3) is:

\[
A_m = d^{-1} \left[\int_0^d \exp\left(2\pi i x (m-b)/d\right) \ dx + p/\exp\left(2\pi i x (m+b)/d\right) \ dx\right] = (2\pi)^{-1} \left[\exp(\pi i (m-b)) - 1\right] (1/(m+b) - p/(m + b))
\]

where \( p = +1 \) for the continuous lens that is shown in Fig. lb and \( p = -1 \) for the lens shown in Fig. lc where the even zones are shifted by \( \lambda \) wavelength. The intensity corresponding to Eq. 14 is

\[
\text{Intensity} = \left(\sin(\pi (m-b)/2)/(\pi (1/(m+b) - p/(m+b)))\right)^2
\]

The intensities given by Eqs. 12 and 15 are tabulated for three diffraction gratings corresponding to the three zone-plate lenses to be considered here:

<table>
<thead>
<tr>
<th>( m )</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. alternating (phase=0, b=1)</td>
<td>0.0018</td>
<td>0.0</td>
<td>0.0450</td>
<td>0.2500</td>
<td>0.4053</td>
<td>0.2500</td>
<td>0.0450</td>
<td>0.0</td>
<td>0.0018</td>
</tr>
<tr>
<td>2. alternating (phase=4, b=1)</td>
<td>0.0288</td>
<td>0.0</td>
<td>0.1801</td>
<td>0.2500</td>
<td>0.0</td>
<td>0.2500</td>
<td>0.1801</td>
<td>0.0</td>
<td>0.0288</td>
</tr>
<tr>
<td>3. nonalternating (b=0.5)</td>
<td>0.0050</td>
<td>0.0083</td>
<td>0.0162</td>
<td>0.0450</td>
<td>0.4053</td>
<td>0.4053</td>
<td>0.0450</td>
<td>0.0162</td>
<td>0.0083</td>
</tr>
</tbody>
</table>

Table 2

The first case is for an alternating zone-plate with the phase shift going from 0 to \( \lambda \) in odd zones and from \( \lambda \) to 0 in the even zones (see Fig. lb). In the second case the phase shift goes from 0 to \( \lambda \) in odd zones and from 0 to \( -\lambda \) in the even zones. This case is equivalent to switching the sign of the amplitude of the even zones (an additional \( \lambda \) wavelength phase, see Fig. lc). In the third case the phase shift goes from 0 to \( \lambda \) in both even and odd zones (see Fig. la). In this last case the blaze coefficient, \( b \), is equal to \( \lambda \). In the alternating cases, however, the distance, \( d \), spans two zones (see Figs. lb and lc) so \( b = 1 \) is the correct value to use in Eqs. 13 - 15.

For the two examples with alternating zones, the intensity for \( m = 1 \) and \( m = -1 \) is 0.25. This occurs because the light into the \( m = 1 \) peak comes only from the even zones, and the odd zones contribute only to the \( m = -1 \) peak. According to Eq. 3, the amplitude is half the original amplitude which causes the intensity to be \( 1/4 \) of the original intensity. Thus 50% of the light goes into the \( m = -1 \) and \( m = 1 \) peaks. The remaining 50% of the light intensity is spread over the other interference orders. The sum of all the intensities is unity. The table show that the nonalternating zone-plate lens has 40.5% of the light being undeviated (\( m = 0 \)) and 40.5% deviated into the first interference sideobe. This particular grating shows the best promise for a contact lens bifocal since it comes closest to the ideal division of the incoming light into two equal parts with 50% intensities.

The axial intensity plot

Fig. 2 presents an overview of the eight optical systems that will be considered. The abscissa
represents the location of a point object. The scale is diopters, corresponding to the inverse of the distance in meters. One must be careful with the sign of the horizontal axis. A +3 diopter lens will focus an object at -3.33 m. Thus a +3 diopter lens will show up as a peak at -3 diopters in Fig. 2. The vertical axis is proportional to \( P(0) \), the intensity of light falling on the retina at the axial point \((x = 0)\).

The width of each peak corresponds to the depth of field whereby a schematic eye with a small pupil will have a larger depth of field than an eye with a large pupil. The depth of field for an eye focussed at infinity can be calculated by using the Pythagorean theorem to calculate the distance at which a marginal ray (the ray from an on-axis object that just touches the pupil stop) travels a wavelength \((/4)\) longer than the axial ray to the center of the pupil. This criterion, called the Raleigh quarter-wave criterion, corresponds to a 20% reduction in intensity of the axial light. The axial ray travels a distance \(D\) and the marginal ray travels a distance \((D^2 + R^2)^{1/2}\), where \(R\) is the radius of the pupil. Setting the difference between these two distances equal to \(\pi/4\) leads to the approximation \(D - 2R/\pi\), corresponding to a dioptric power of \(1/2R\). For \(R = 0.56\) microns and a pupil radius of \(r = 1.7\) mm, the depth of field (corresponding to 80% intensity) is approximately \(1/2R \approx 0.07\) diopters. This remarkably small value accounts for the narrow peaks in Fig. 2 and for the rapid degradation of the MTF (to be calculated) as the object point deviates from the far point.

A surprising feature of the two-zone lens (Fig. 2b) is that there are three major peaks rather than the expected two peaks at 0 and -3 diopters. The two outside peaks are shifted to 0.35 and 1.45 diopters. The largest peak, at -1.5 diopters corresponds to constructive interference between the two zones. The coherence between the two zones is discussed next in connection with the 8-zone alternating lens. The dioptric width of the two-zone peak is larger than that of one of the one-zone peak because of the inverse square dependence on pupil size discussed in the preceding paragraph. A larger depth of field sounds like an advantage, but as will be seen, the larger depth of field is associated with a larger blur circle, resulting in a loss of high spatial frequencies.

The Fresnel lens (Fig. 2c) in which each of the eight zones is given a random phase shift shows a complex interference pattern. If all eight zones had been added coherently the one-zone curve shown at the bottom of Fig. 2 would have resulted. The random phases that are present in Fresnel optics are to blame for the poor image quality that is indicated in Fig. 2 (see also Figs. 7 and 8).

The zone-plate alternating lens with phase \(= 0\) (Fig. 2d) is similar to the two-zone lens except now there are 3 zones, with the innermost zone having a .5 mm radius rather than 1.2 mm in the 2-zone lens. According to geometrical optics the alternating zones of 30 and 53 diopters should have had two focal points corresponding to 0 and -3 diopters. However, three focal points are seen at 10, 1.5, and -3 diopters, similar to the 2 zone case. This multizone lens would do well as a trifocal rather than a bifocal. The extra focal point will now be justified. The location of the cornea is approximately given by \(x = y^2/2R\) (see Eq. 11). The additional optical path length due to the curvature is

\[
L = (n-1)x = \left(\frac{y^2}{2}\right) \left(\frac{n-1}{R}\right) = \frac{y^2}{2R}
\]

since the dioptric power is \(D = (n-1)/R\). For the \(k\)th zone, \(y = 0.6k\) so the difference in \(x\) at the zone boundaries between the 30 and 53 diopter zones is

\[
L = \frac{y^2}{2} \Delta 0/2 = (0.6k)^2 \left(\frac{53 - 50}{2}\right) = 0.56k (1.5) = 0.84k \text{ microns}
\]

Since the wavelength of light is .56 microns each transition is almost exactly one wavelength. If each zone is shifted by one additional wavelength to eliminate the sharp edge (as suggested in Cohen's patent) then the average radius of curvature of the cornea corresponds to 3.15 diopters, and it is not surprising to find zone-plates, where a calculation of the intensity of the three peaks is presented. The zone locations (.6k) for this example do not correspond to the optimal spacing suggested by Cohen. The Cohen family of lenses will be examined in a separate publication.

The 8-zone alternating lens in which the even zones (53 diopter) have an added half-wavelength phase shift (Fig. 2e) does not show the peak at -1.5 diopters. Rather the side peaks at -1.5 and 1.5 diopters are enhanced. This effect of the phase shift is in agreement with the previous discussion about the peak at +1.5 diopters being due to destructive interference between the even and odd zones. The peak disappears when the even and odd zones are destructively.
The zone-plate nonalternating lens (Fig. 2a) uses zone spacings that are identical to the alternating zone-
plate as given in Eq. 17; \( y^2 = 2v \lambda a \) where \( aD \) = 5.5, 50 = 3 diopters in the present case. Fig. 2 shows two
major peaks at -0.02 and -3.08 diopters. The shift from -0.02 to -3.08 diopters will be relevant to the plots in
Fig. 2. In addition, small peaks are seen at ±5 and ±3 diopters. The origin of these additional peaks will
be discussed in the section on blazed zone-plates. Between the two major peaks are six small peaks
reminiscent of the interference pattern that results from light passing through eight linear slits.

The presence of spherical aberration (Fig. 2b) shifts the peak leftward to -0.19 and -3.38 diopters rather
than -0.02 and -3.08 of the zero aberration basic lens as we previously discussed. This is not surprising since
the spherical aberration of a spherical lens causes the dioptic power to increase away from the center of the
pupil. The spherical aberration did not significantly increase the width of the dioptic peaks because the
pupil diameter of 3.4 mm is small enough so that the broadening due to spherical aberration is not larger than
the broadening due to diffraction.

The top plot in Fig. 2 is obtained by using "white light" with the zone-plate lens. Five wavelengths that
roughly span the photopic sensitivity curve were used. The multiple wavelengths had minimal effect on the
peak at zero diopters, but it did shift the peak at -3 diopters and almost eliminated the minor side peaks at
±6 and ±3 diopters.

The remainder of this paper will examine the point spread functions and modulation transfer functions of
the eight lenses shown in Fig. 2.

Image quality of the eight lenses

1. The one-zone schematic eye. Figs. 3 and 4 show point spread functions, and modulation transfer functions
(MTF) for the schematic eye (one-zone) described earlier. Each figure has 10 plots corresponding to dif-
frent object distances. The far group has object locations of ±2.5 m, ±5 m and infinity (corresponding to
±4, ±2 and 0 diopters of incoming vergence). The near group object locations are ±267, ±30, ±333,
±367, and ±40 m (corresponding to ±3.75, ±3.33, ±2.73, and ±2.5 diopters of incoming vergence). The
far and near groups are centered around infinity and ±333 m, the two far points of the bifocal lenses.

The vertical axis in Fig. 3 is not simply the point spread function, \( P(r) \), representing the light intens-
ity at a given retinal point. Rather, the plots show the intensity times the radial distance, \( r \), from the
axis. There are two reasons for the extra factor of \( r \). First, the total amount of light falling at radius
\( r \), is \( P(r) r \ dr \), where \( P(r) \) is the point-spread function (Eq. 4), and \( r \ dr \) is the area of a thin ring at
radius \( r \). The quantity \( r P(r) \) that is plotted in Fig. 3 is a measure of the amount of light at different
radii. This is the quantity that appears in the calculation for the modulation transfer function (Eq. 5).
The second reason for plotting \( r P(r) \) is for cosmetic reasons. If \( P(r) \) had been plotted, then the axial
value (x=0) of the point spread function, \( P(0) \), for the in-focus image would be so much larger than the off-
axis values that the latter would not be visible on the plot. The factor of \( r \) reduces the central peak and
enhances the annular region so that the full significance of the entire point spread function can be readily
viewed.

The outside radius of the blur circle can be calculated by geometrical optics in terms of the pupil radius
and diopters of defocus. The result is

\[
\text{blur} = 0D/n \text{ (radians)}
\]  

For \( p = 1.7 \) mm, \( D = 3 \) diopters and an index of refraction of \( n = 1.5 \), the blur circle has a radius of 11.7
min on the retina, in agreement with Fig. 3. In object space the blur function is \( r \text{blur} = 0D \). This simple
result means that for an eye focused at infinity, the blur size of an object at a particular distance equals
the angular subtense of the pupil at that distance.
Figure 4 shows plots of MTF(f), providing a clear measure of image quality that is complementary to the information in the point spread function. The optical MTF is always normalized to unity at zero spatial frequency, so that the total light leaving the lens equals the total light entering the lens (no absorption). The frequency (in radians/min) at which the MTF falls to 1/e (37%) gives a rough indication of the halfwidth of the point spread function. In Fig. 3, the 37% point is about 60 c/deg (or equivalently, 2 rad/min) corresponding to a point spread function 1/(π(π + s^2)) where the space constant s is (2.5^2) = .15 min. The MTF is useful for showing what information is lost. For example, an object that is 2.5 m in front of the eye will have all information above 20 c/deg reversed in sign (spurious resolution).

2. The two-zone concentric bifocal. The two-zone concentric design has an inner diameter of 2.4 mm and an outer diameter that is 21 larger (3.39 mm). The ratio of inner and outer diameters was chosen so that an equal amount of light would go through each zone. The outer diameter of 3.39 mm is in reasonable agreement with the size of the natural pupil. The two dioptric powers for the two zones were chosen to be 50 and 53.0. The 50 D central zone brings an object at infinity to a focus on the retina of the schematic eye. The 53.0 annular zone brings an object at -333 mm to focus. The value of 53.0 was chosen to be in approximate agreement with the power of the nonalternating zone-plate bifocal that was calculated.

Fig. 5 and 6 show point spread functions and MTFs as in Figs. 3 and 4. The easiest point spread function to explain is for the case when the central zone is in-focus and the outer zone is out-of-focus (top half of Fig. 5). In this case there is no overlap between the images from the two zones since the light from the outer zone falls in an annular ring on the retina. As the object distance deviates more from the far point, the radius of the blur annulus increases. The location of the blur annulus can be calculated by ray tracing as in Eq. 18. The point spread function becomes more complicated when the central zone is out-of-focus, since now the light from both zones overlap (both the inner and outer zones send light to the axial region) and a complex interference pattern results.

Fig. 6 shows the MTF plots corresponding to the point spread functions of Fig. 5. Not surprisingly, the MTFs for object distances of infinity and -333 mm are similar to the plots for other object distances. One of the characteristic features of any bifocal design is that the MTF decreases rapidly at low spatial frequencies to about 50% of the maximum value that is seen in Fig. 6. This is characteristic of all "equivalent" bifocal contact lenses. The attenuation is due to the blurred light from the out-of-focus image lowering the in-focus contrast. For the 2-zone lens there is additional compression along the spatial frequency axis. For example, the cut-off spatial frequency is about 100 c/deg as compared to 200 c/deg for the one-zone lens (Fig. 4). The reduction in cut-off spatial frequency by 1.4 occurs because the inner zone radius of the two-zone lens is smaller by a factor of 1.4.

The annular portion of the two-zone lens is not as easily understood. The point spread function for the case of an object at -267 mm, has sharp peaks at 1.25, 2.25 and 3.25 mm, and then a broad peak at about 8 min. The MTF of the light that comes from the annulus (the region focussing on close objects) is quite inferior to the MTF for the central zone. That is, for an object at -333 mm, Fig. 6 shows that a 15 c/deg stimulus would have its contrast reduced to about 15%.

Detailed calculations were also made for the "inside-out" two-zone lens with 90 diopters in the center and 50 diopters in the annulus. The plots are not shown because they are identical to Figs. 5 and 6 except that the near and far object locations must be switched. That is, the MTF of the 53-50 lens for an object at infinity is the same as the MTF of the 50-53 lens for an object at -333 mm.

2. Multiple zones and the Fresnel lens. The remainder of this article explores multizone lenses. Before constructing coherent bifocal designs we will examine the multicomponent Fresnel lens. The Fresnel lens is the most familiar multiple zone lens. It is commonly found, for example, on overhead projectors and rear window of buses. Fresnel optics for ophthalmic use have been manufactured by MM Corp. An example is the
Fresnel prism that can be pressed on the back of spectacles to compensate for an eye turn. Fresnel lenses however, have not succeeded in the marketplace. We must examine the failure of the Fresnel design in order to understand why the zone plate design should work better.

A Fresnel lens or Fresnel prism can be thought of as a regular lens or prism that is divided into multiple sections and then made flat while preserving the surface tilt. Two complaints about Fresnel spectacles are:
1. The image quality is degraded by about one line of visual acuity and by about a factor of 2 in peak contrast sensitivity. 2. The spectacles look cloudy when seen by a distant observer.

It is likely that these problems stem from the fact that the light rays from the different zones are out-of-phase with each other. In a Fresnel lens the height of each discontinuity is not accurately controlled so that the light passing through each zone has a random phase relative to the light from any other zone. If light from N different zones is imaged onto a retinal point, then the expected amplitude of the axial light would be decreased by N\(^{-1}\) and the intensity (amplitude squared) would be decreased by N. Conservation of energy requires that the decreased on-axis intensity results in an increased off-axis intensity, resulting in an enlarged point spread function (reducing acuity) and in diffuse scattering (reducing contrast sensitivity and producing the cloudy appearance when a Fresnel lens is viewed from far). Figs. 7 and 8 show the point spread function and the modulation transfer function of an eight-zone Fresnel lens.

The multiple zone lenses that are the topic of this paper do not have the problem of random phases because the phase shift of each zone is precisely controlled. In order to have the light from each zone add coherently, the phase shifts should be accurate to better than 10\% of the wavelength of light. This tolerance of \(0.05\) microns is quite feasible with modern technology. The remainder of this paper assumes these levels of accuracy for multizone lenses.

4. The zone-plate alternating lens (phase = 0). Fig. 1b showed the phase shift pattern of this zone-plate alternating lens. The focal point locations were shown in Fig. 2c, where it can be seen that this lens would make a decent trifocal. Its image quality is shown in Figs. 9 and 10. Both the point spread function and the MTF indicate that the image quality is poor for the particular lens investigated. For the in-focus cases of an object at infinity and at \(-.333\) m, the point spread function shows distinct side lobes at about 5 min that are almost as large as the central peak. The MTF shows a rapid decrease to about .25 at 5 c/dim, rather than a decrease to .5 that would be wanted in an ideal bifocal.
5. The zone-plate alternating lens (phase = i). Fig. 1c shows the phase shift pattern for a lens in which the phase of the even zones are shifted by \( \frac{1}{2} \) wavelength compared to the preceding lens (see discussion following Table 2). Fig. 2 showed that the lens is expected to have two primary focus points each of which focuses \( i \) of the incoming light. The image quality of this lens as shown in Figs. 11 and 12 is fair. Since \( \frac{3}{4} \) of the light for a particular image has been lost, it is hard to claim that the image quality is very good.

6. The zone-plate nonalternating lens. Figs. 13 and 14 show the point spread functions and MTFs for the nonalternating zone-plate lens. The blaze scheme for a circular lens is slightly different from the linear diffraction grating discussed earlier in connection with Eq. 12. For the zone-plate nonalternating lens, the following phase shift was added to the light in the \( k \)th zone

\[
\text{Phase} = b 2\pi \left( \frac{r}{r_0} \right)^2 - k \text{ radians}
\]

where \( r_0 \) is the radius of the first zone. Eq. 19 gives a phase shift of 0 at the beginning of each zone and a phase shift of \( b 2\pi \) at the end of each zone, since the \( k \)th zone begins at \( r = r_k \). A value of \( b = 0 \) correspond to a 50 diopter cornea, and a value of \( b = 1 \) corresponds to a 53 diopter cornea. The results of Figs. 13 and 14 are for the intermediate blaze coefficient of \( b = 0.5 \).

As might be expected from Fig. 1, the image quality is good for both an object at infinity and an object at \(-333\) m. The steep falloff at low spatial frequency extrapolates to an intensity of about 0.45, which is better than all the other lenses investigated so far.

In order to fully appreciate the effect of the blaze coefficient, Figs. 15 and 16 show the point spread functions and MTFs for blaze coefficients of \( b = 0, 0.2, 0.4, 0.5, 0.6, 0.8 \) and 1.0. The MTF curves for object locations of infinity and \(-324\) are shown. The object distance of \(-324\) was used for two reasons: First, the peak focal point in Fig. 2f occurred at \(-308\) whose reciprocal is \(-324\). Second, by examining curves similar to those in Fig. 14 but with finer object position separations we found \(-324\) was optimal. The point spread functions are shown only for an object at infinity since as seen in Fig. 16, the object locations of \(-324\) and infinity are interchangeable when \( b \) is replaced by \( 1-b \). One of the distinctive features of the MTF for the zone-plate is the oscillations with a period of about 9 c/deg that are seen in Fig. 16. The same oscillations are found for a purely out-of-focus object as seen in Fig. 4 for a one-zone 50 diopter lens (the simple schematic eye) for an object at \(-333\) m. The corresponding point spread function is seen to have a peak at about 9 min on the retina. One could worry that the peak at 5 c/deg could be disturbing to the con-
contact lens wearer, but the fairly flat point spread function indicates that the background blur circle would be innocuous. We plan to convolve Snellen letters with the various point spread functions calculated in this paper to determine by psychophysics the extent of acuity loss caused by the different designs.

7. Effect of spherical aberration. All the preceding lenses were calculated with an aspheric cornea chosen to have minimal spherical aberration (see Eq. 1). The effect of spherical aberration can be examined by using a spherical cornea. Figs. 17 and 18 show the image quality of a spherical cornea together with the monochromatic zone-plate contact lens. The main effect of the spherical aberration is to shift the curve as if the eye were myopic just as was shown in Fig. 2. Fig. 18 shows that image quality for an object at -5m is as good as the image quality from the zero spherical aberration lens (Fig. 14) for an object at infinity. Spherical aberration doesn't cause severe degradation of an optimally focussed image unless the pupil is larger than the 3.4 mm pupil used in the present study. One unexpected feature is the dramatically rapid image degradation that occurred in going from an object distance of -0.300m to -2.67m.

8. Effect of multiple wavelengths. A possible problem with zone-plate optical elements is that the zone spacing for constructive interference depends on wavelength. From Eq. 17 it is seen that the kth zone boundary should be at \( y = (2k\lambda / AD)^{1/2} \), where \( \lambda \) is the wavelength of light and AD is the desired difference in diopters. To investigate possible image degradation due to multiple wavelengths we used "white light" con-
sisting of 5 wavelengths described earlier. Figs. 19 and 20 show the image quality for our "white light" stimulus. The lens used was again the zone-plate nonalternating design. Only two object positions are shown one for near and one for far viewing. The main effect of using a broad spectrum of light is that the near focus moves out from –.353 m to –.373 m. A blaze coefficient of .55 was used rather than .5 since it was found to give the best balance in MTF quality between the near and far object positions. The image quality with the five wavelengths was found to be good for both near and far viewing. Both the point spread function and the MTF are smoother than is the case when monochromatic light is used (Figs. 13 and 14). Note that especially for the close object position the MTF ripples are eliminated. The excellent image quality of the luminance point spread function and luminance MTF is a bit deceptive since there will be chromatic fringes that we have not examined.

Discussion

We consider the present work to be just a beginning. Yet to be done is the laborious task of finding a phase profile for a zone of the zone plate that will provide optimal image quality for a bifocal contact lens. The present study leads us to believe that successful bifocal contact lenses are feasible. During the next year we anticipate finding designs that are a bit better than those discussed so far. In searching for improved designs it is difficult to rely on one's intuition. This is because the human eye is operating near the limits of diffraction, spherical aberration and chromatic aberration. In this regime the dualistic wave-particle nature of light becomes important. Geometric optics provides the broad characteristics of the blur circle, but interference effects produce ripples that can be quite annoying and can degrade good vision. It remains to be seen whether the ripples are bothersome. We intend to convolve Snellen letters with appropriate point spread functions in order to learn about the loss of image quality of real images. Other projects for the coming year are to examine the image quality for decentered pupils, for improved schematic eyes and for off-axis objects. We are optimistic that successful lenses will be found.

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References


