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"WEBER'S LAW" FOR POSITION: UNCONFOUNDING THE ROLE OF SEPARATION AND ECCENTRICITY

DENNIS M. LEVI, STANLEY A. KLEIN* and YEN LEE YAP†
University of Houston, College of Optometry, 4901 Calhoun Blvd, Houston, TX 77004, U.S.A.

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Abstract—Bisection thresholds are approximately proportional to the separation/eccentricity of the targets. This "Weber's law" for position has been invoked over the past century. Is it the separation of the reference targets, or their eccentricity which determines the threshold? In previous studies separation and eccentricity are confounded. In the present report we have pitted separation against eccentricity. Bisection thresholds were measured for stimuli presented on an isoeccentric arc, so that separation could be varied while holding the eccentricity of the test lines constant. We used a 5-fold range of separations from 2–10 deg. In this regime, the present results provide strong evidence *against* Weber's law. When separation is varied but eccentricity held constant, there is no Weber's law. Rather the thresholds are approximately constant. Our results suggest that the judgement of the separation of widely separated objects is similar to a distance measurement using a ruler on the cortex, in that the error of measurement is independent of the separation between objects. The results imply that when we attempt to gauge the distance between widely separated objects it is unlikely that we do so on the basis of the outputs of large spatial filters; rather it appears that we make such judgements by estimating the cortical distance which separates the targets of interest.

Periphery Psychophysics "Weber's Law" Position discrimination Separation discrimination

INTRODUCTION

Human observers can judge the relative position of targets with exquisite accuracy. While positional acuity is often thought of in terms of the very precise spatial thresholds which are obtained when the target features are close (Westheimer, 1975), one of the most striking features of positional acuity, is that over a wide range of conditions, the position threshold is approximately proportional to the separation/eccentricity of the reference features. Over a century ago, Fechner (1858) and Volkmann (1858) asked observers to judge the position of a "test" line relative to a pair of flanking reference lines. The results of these bisection experiments showed that the mean errors obtained in bisecting a space were approximately a constant fraction of the distance being bisected. This "Weber's law" for position has been frequently replicated (Hirsch and Hylton, 1982; Levi and Klein, 1983; Bedell *et al.*, 1985; Klein and Levi, 1985, 1987; Toet *et al.*, 1987; Yap *et al.*, 1987a,b; Burbeck, 1987). Under "ideal" conditions, i.e. long durations or long lines, when thresholds are plotted as a function of the separation of the reference lines on log–log coordinates, the resulting curve is a straight line with a slope of 1. When both the spatial and temporal stimulus conditions are not ideal, i.e. brief durations and short lines, the optimal thresholds at small separations are elevated, and thresholds follow a power function of separation with an exponent of about 0.7 (Yap *et al.*, 1987a).

There are two general classes of explanation for this Weber's law for position: one is based upon the differential responses of spatially localized filters (e.g. Klein and Levi, 1985; Wilson, 1986). As the separation between the test line and the flanks increases, larger filters (with zero crossings approximately equal to the line spacing) would be used, giving proportionately higher thresholds. However, for large separations this approach becomes less likely, since the filters would have to be implausibly large and maintain high sensitivity. (Weber's law holds to at least 10 deg). The second approach is based upon the spatial grain of the visual system (Hering, 1899; Matin, 1972; Burbeck,

*Present address: School of Optometry, University of California at Berkeley, CA 94720, U.S.A.
†Present address: Visual Sciences Program, Sensory Sciences Research Laboratory, SRI International, Menlo Park, CA 94025, U.S.A.

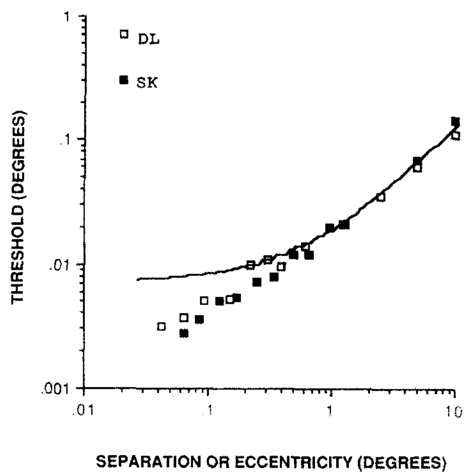

Fig. 1. Bisection thresholds versus separation/eccentricity for two of the observers in the present experiments are replotted here in log–log axes from the data of Klein and Levi (1987). The curve represents their large eccentricity fit to the data: i.e. in linear–linear coordinates it is a straight line which intersects the x-axis at -0.6 deg. In the log–log coordinates shown here, it (1) provides a good fit to the data for eccentricities >0.5 deg, and (2) exhibits Weber's law behavior at large eccentricities, i.e. it becomes a straight line with a slope near 1.

1987; Klein and Levi, 1987). The notion here is that the spatial grain of the cortex (M^{-1}) varies linearly with eccentricity*; thus the position of the reference lines becomes increasingly uncertain as their eccentricity increases.

Recently we (Klein and Levi, 1987) measured position acuity using a bisection task. In this experiment, a central reference line appeared, and was followed by a pair of briefly flashed peripheral "test" lines. We used the bisection task to determine the precision with which the observer could judge the position of the peripheral test lines, with only a foveal reference. The data for two observers are replotted in Fig. 1 in log–log coordinates, and show the proportionality between thresholds and separation/eccentricity over a 200-fold range from 0.05 to 10 deg. The curve in Fig. 1 is given by $\text{Th(deg)} = 0.012(1 + \text{Ecc}/0.6)$, and will be discussed further below. Our analysis, based upon these results suggested that both classes of explanation may be correct. For separations/eccentricities less than about 15–20 min, the data were compatible with predictions based upon the outputs of localized filters; however, for larger separations/eccentricities the position thresholds appear to be set by the spatial sampling grain of the cortex. In this regime, thresholds appear to represent a *constant cortical distance* (about 0.1 mm). In other words, for eccentricities larger than 15–20 min, there is no Weber's law for position on the cortex. This large separation/eccentricity regime is important to our understanding of distance judgements, and is the subject of the present report. A strong prediction of the spatial grain hypothesis, is that in the periphery, only the eccentricity of the stimuli and *not* their separation should matter†. While previous experiments on bisection in the periphery (e.g. Yap *et al.*, 1987a) show that bisection thresholds depend upon separation, they do not provide a clear test of this notion, since eccentricity and separation are confounded at large separations. This report describes an experiment designed to unconfound the role of eccentricity and separation.

*Bisection thresholds, T, vary with eccentricity, E (deg), according to $T = T_f(E/0.6 + 1)$ where T_f is the foveal threshold (Yap *et al.*, 1987a).

†We are indebted to Jim Bergen for pointing out that "any fool can see the implication of this hypothesis". We, not being any fool, had not seen it!

METHODS

The stimuli consisted of three bright vertical lines generated on the screen of a computer (Commodore Pet). The lines were approx. 30′ long and 3′ wide at the viewing distance of 39 cm. Briefly, a vertical "reference" line was presented for 1 sec. A pair of flanking "test" lines, aligned horizontally with the reference line were then presented, and 200 msec later all three lines were extinguished. The observer's task was to make a bisection judgement about the direction and magnitude by which the reference line appeared to be displaced from the midpoint of the peripheral test lines. The horizontal position of the entire display was randomly jittered from trial to trial by approx. 1.8 deg to eliminate absolute position cues. Threshold for each experimental run of 125 trials was obtained by a maximum-likelihood multiple criterion probit analysis of the data. The threshold corresponds to the offset giving a $d' = 0.675$ (75% correct). The thresholds reported here are the weighted geometric means of 3–9 runs per condition, and the error bars include both within and between session variability. Further details of the experimental methods are given in Klein and Levi (1987).

In order to disambiguate separation and eccentricity, the peripheral location and sepa-

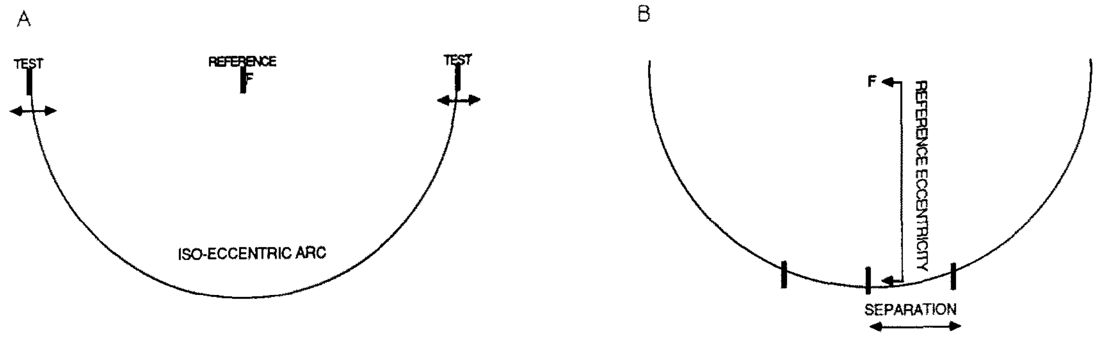

Fig. 2. Schematic illustration of the stimulus arrangement. The stimuli consisted of three lines. A reference line was turned on and 1 sec later, a pair of flanking test lines were briefly presented. The test lines were always centered on an isoeccentric arc of 10 deg. Thus the eccentricity of the test lines remained fixed while the eccentricity of the reference line (the "reference eccentricity" shown in the right panel) varied in the lower visual field. This allowed a 5-fold variation in the separation of the test targets, while maintaining a constant eccentricity.

rations of the stimuli were varied from run to run. Figure 2 illustrates two examples of the stimulus arrangement. The two test lines always fell on an isoeccentric arc, 10 deg from the fovea. In this way the mean separation of the test and reference lines varied between 10 deg (Fig. 2A) and 2 deg (Fig. 2B) while the eccentricity is fixed at 10 deg. Note that the position of the vertical test lines fell on the 10 deg isoeccentric arc (plus or minus a small offset corresponding to the bisection cue) with a horizontal chord, while the eccentricity of the reference stimulus varied. When the reference line was at the fovea (as shown in Fig. 2A) the observer fixated it directly. When the reference line was presented in the lower visual field (Fig. 2B) the observer was instructed to fixate directly above the reference line, along a horizontal strip placed at the appropriate height. In this way, under all conditions, a single test line was presented to each hemifield. Two possible outcomes for this experiment are illustrated in Fig. 3, which shows schematically thresholds plotted against separation on log–log coordinates. If separation is critical then a straight line with a positive slope will obtain (Weber's law is represented by a slope = 1); however, if the spatial sampling grain hypothesis is correct, then a line with a slope = 0 will obtain, since the eccentricity of the test stimuli is always 10 deg.

RESULTS

The 3 authors, each with corrected to normal vision, served as observers and their results are shown in Fig. 4. The open symbols show thresholds for a 5-fold range of separations (shown on the lower abcissa) with the reference line presented at the eccentricities shown on the upper abcissa. The Y, S and D show thresholds for the 10 deg separation with the foveal reference (Fig. 2A). The results of each of the observers offers clear evidence for an *absence* of Weber's law. Surprisingly, the thresholds change only slightly, and for separations greater than 3.5 deg they change in the opposite direction from the Weber's law prediction, i.e. they decrease rather than increase with increasing separation.

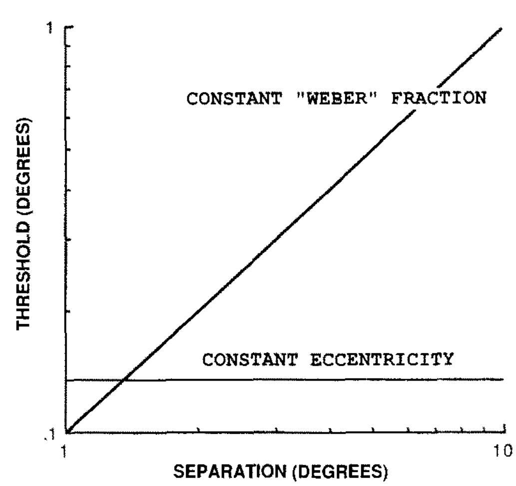

Fig. 3. Illustrates the predictions of two classes of models: (1) if Weber's law for position holds, i.e. if thresholds are proportional to the *separation* of the target feature, then on log–log coordinates, the thresholds shoud fall along a line with a slope of 1. (2) On the other hand, the spatial sampling grain hypothesis holds that thresholds are a constant fraction of the eccentricity of the test lines, yielding a line with a slope of 0.

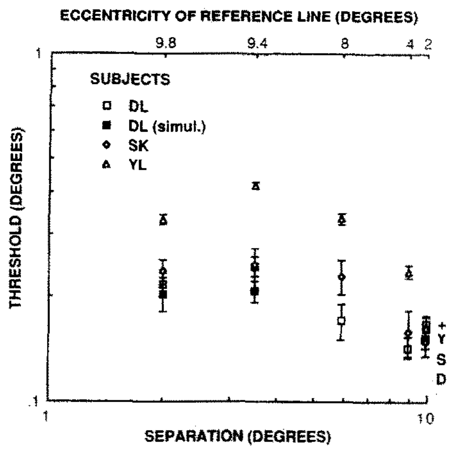

Fig. 4. Thresholds (in degrees) are plotted as a function of the separation (in degrees, shown on the lower abscissa) of the pair of test lines for three observers. The open symbols show the conditions where the reference line was in the lower visual field, at the eccentricities indicated on the upper abcissa. The letters Y, S and D show the data when the reference line is directly fixated (Fig. 1 left) for each observer (the letters indicate the first initial of each observer). The results clearly violate Weber's law. However, they do not conform perfectly to a line with a slope of 0 (the spatial grain or constant eccentricity hypothesis). The results of control experiments with simultaneous presentation of the three lines are shown for observer D.L. by the filled squares, and are similar to the data obtained with asynchronous presentation (open squares) except for the condition with the foveal reference (+ for simultaneous, D for asynchronous).

One possible reason for the elevated thresholds at small separations is metacontrast masking. For separations of 2 and 4 deg, each of the observers noted that the flashed test target appeared to produce a marked attenuation of the brightness of the stationary reference line. In order to test whether this perceptual masking might have influenced thresholds, we repeated the experiments with all 3 dots flashed simultaneously (for 0.2 sec). This eliminated the perceptual masking, but did not significantly influence the position thresholds. The filled squares show the thresholds for the simultaneous onset condition for observer D.L. at separations of 2, 3.5 and 9.9 deg. The data are essentially identical to those obtained with the asynchronous onset (shown by the open squares). The only case in which simultaneous and asynchronous thresholds differed was for the condition in which the reference was presented to the fovea (Fig. 2A). In this case the asynchronous condition allows precise foveal fixation of the reference position. Thus it is not surprising that thresholds for this condition were about 30% better for the asynchronous than for the simultaneous condition, since the 1.8 deg jitter and brief duration made precise foveal fixation of the center line impossible. For D.L., thresholds for the asynchronous condition (shown by the D) was 0.12 deg, and for the simultaneous condition (shown by the +) it was 0.16 deg. Thus, it is unlikely that the shape of the curve can be explained on the basis of metacontrast masking.

We have made similar measurements with only a single test line. The stimuli and methods were identical to the first experiment, but with only a single vertical "test" line rather than a line in each hemifield. The observers judged the separation between the test and reference lines relative to a memorized standard rather than making a bisection judgement as in the 3-line task. The open symbols in Fig. 5 show the data of D.L. and Y.L.Y. for separations of 3.5 and 10 deg. For both observers, the results are qualitatively similar to those obtained with the test line in each hemifield, with thresholds decreasing slightly with increasing separation.

While the thresholds shown in Figs 4 and 5 are not entirely independent of separation, they

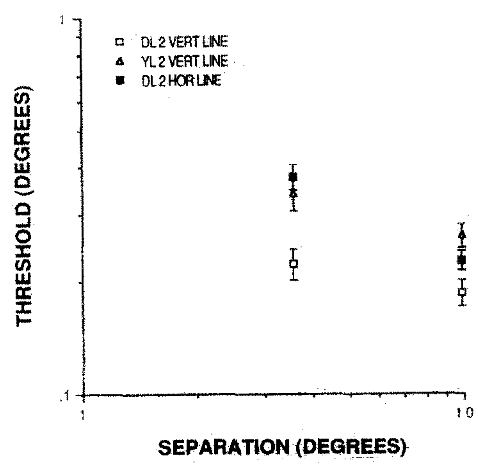

Fig. 5. Thresholds (in degrees) are plotted as a function of the separation (in degrees) of the *single* test line. The stimuli and methods were identical to the first experiment, but with only a single vertical "test" line rather than a line in each hemified (i.e. the observer judged the distance between the test and reference lines). The open symbols in Fig. 5 show the data of D.L. and Y.L.Y. for separations of 3.5 and 10 deg. The filled symbols show the results of a control experiment in which the reference line was horizontal, and a single horizontal test line was presented along a new isoeccentric arc with its chord vertical, and extending into the left visual field. This stimulus arrangement can be envisaged by rotating Fig. 2 by 90 deg clockwise. This arrangement places the 10 deg separation on the low acuity meridian, while the small separation falls along the high acuity meridian. Note that for observer D.L. (squares), thresholds for the horizontal test line were higher than with the vertical test line for *both* the large and small separations.

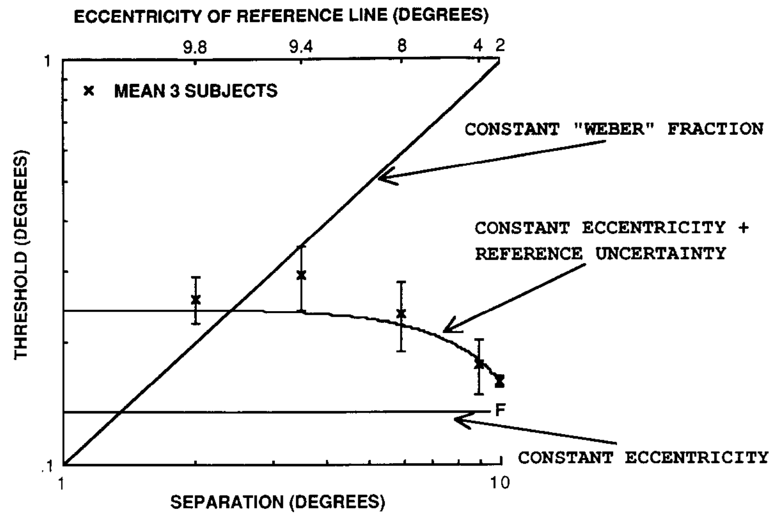

Fig. 6. This figure summarizes the main results by plotting the geometric mean of the thresholds (×) for the three observers for separations between 2 and 10 deg with the reference line presented at the eccentricity shown by the upper abscissa. The error bars reflect the between-subject standard deviation. The 'F' shows the mean threshold when the reference was directly fixated (Fig. 2A). The data clearly violate the Weber's law prediction. However, they do not fit the constant eccentricity prediction either. Rather, the data are reasonably well fit by assuming that the positional uncertainty of the peripheral reference is approx. 1% of the reference eccentricity, and that this uncertainty adds linearly to the constant eccentricity prediction as shown by the curve marked "constant eccentricity + reference uncertainty".

change only slightly, and for separations greater than 3.5 deg, in the opposite direction from the Weber's law prediction, i.e. they decrease rather than increase with increasing separation. Could this surprising decrease in thresholds with separation be a consequence of retinal or cortical orientational anisotropy in that for wide separations the test lines are offset in the radial direction while for small separations they are offset along an isoeccentric direction? The present data cannot be explained on this basis because bisection thresholds in peripheral vision have been shown to be worse when the stimulus offset is along a line which is radial to the fovea than when it is tangential to it (Yap *et al.*, 1987b). This is contrary to the present results since thresholds for small separations, where the direction of offset was tangential were worse than those for large separations, where the offset was in the radial direction.

An alternative consideration is one based upon the meridional anisotropy of acuity. Our stimuli were presented on an isoeccentric arc; however the isoacuity contours are elliptical, with acuity in the horizontal meridian being somewhat better than in the vertical meridian (Wertheim, 1894). Thus, it is possible that when our stimulus separation was 10 deg, it fell on a region of high acuity (along the horizontal meridian), whereas the small separations fell upon a region of lower acuity. Since this could account for thresholds being worse at small separations, we performed two control experiments with the display rotated by 90 deg. For control experiment 1, the reference line was horizontal, and the pair of horizontal test lines were presented along a new isoeccentric arc along a vertical chord in the left visual field. This stimulus arrangement can be envisaged by rotating Fig. 2 by 90 deg clockwise. This arrangement places the 10 deg separation on the low acuity vertical meridian, while the small separation falls along the high acuity horizontal meridian. For D.L. the threshold for a separation of 10 deg along the vertical meridian was 0.156 ± 0.015 deg, (slightly worse than for the same separation along the horizontal meridian); however, for a separation of 3.5 deg along the high acuity, horizontal meridian, threshold was 0.50 ± 0.05 deg. Similar results were obtained in control experiment 2 with a single horizontal test line and these data are presented in Fig. 5 (filled squares). Note that for observer D.L. (squares), thresholds for the horizontal test line (arranged vertically) were higher than with the vertical test line (open squares) for *both* the large and small separations. However, even with the small separation presented along the high acuity horizontal meridian, and the large separation along the low acuity vertical meridian, there is no evidence for Weber's law, and the slight decrease in thresholds with increasing separation is still apparent. Thus, these control experiments suggest that the shape of the function cannot be attributed to differences in acuity.

Figure 6 summarizes the data for the three line, asynchronous condition (Fig. 4) by plotting the geometric mean of the thresholds of the three observers. It is clear that these data do not conform to Weber's law. Rather, we believe that these results are compatible with the spatial grain hypothesis, and that there is a simple explanation for the non-zero slope to be discussed next.

In these experiments, because the middle reference line is more central than the test lines, its position uncertainty is less than the uncertainty of the test lines. When the reference line is presented to the fovea (Fig. 2A and the "F" in Fig. 6) the positional uncertainty of the reference will be negligible; however, as separation decreases, the entire stimulus array is shifted into the lower visual field, so that the reference line is presented at increasingly more eccentric loci. Thus, it is likely that the positional uncertainty of the reference increases as separation decreases. We have previously argued that the positional uncertainty of the visual system is approx. 1% of effective eccentricity (i.e. eccentricity +0.6 deg). This is equal to about 0.1 mm of cortex at all eccentricities if we assume that human foveal magnification is approx. 15 mm/deg (Cowey and Rolls, 1974), and varies with eccentricity at about the same rate as does cortical magnification in monkeys (Dow *et al.*, 1981; Tootell *et al.*, 1982; Van Essen *et al.*, 1984) and positional acuity in humans (Levi *et al.*, 1985; Klein and Levi, 1987; Yap *et al.*, 1987a). In the fovea, this would be about 0.006 deg, and would increase in peripheral vision according to $P = 0.006\ (\mathrm{ecc}/0.6 + 1)$ where P is the positional uncertainty at a given eccentricity, ecc. Thus, for the stimulus with a reference at 2 deg (separation of 9.8 deg), the uncertainty would be about 0.03 deg, whereas for a separation of 2 deg, the reference would be 9.8 deg from the fovea, and would have a positional uncertainty of about 0.11 deg. The curve in Fig. 6 marked "constant eccentricity + reference uncertainty" shows the influence of linearly adding the positional uncertainty of the reference to the constant eccentricity prediction. The results provide a reasonable fit to the data (Pythagorean summation does not provide quite as good a fit), and suggest that for large separations, there is no Weber's law for position. Rather, thresholds are equal to a constant plus an additive positional uncertainty of the reference.

DISCUSSION

Weber's law for position has been invoked over the past century. In previous studies we (Klein and Levi, 1987) measured bisection thresholds over a range of stimulus separations/eccentricities from 3 min to 10 deg. Our results suggested that the bisection data required a two-line fit. For eccentricities less than 15–20 min, thresholds were in good agreement with calculations based upon the differential outputs of localized spatial filters; however, at larger eccentricities, the results were consistent with the spatial grain hypothesis. On linear axes, the line segment at eccentricities >0.5 deg had an x-axis intercept of about 0.6 deg, in agreement with previous estimates of the cortical magnification factor (Levi *et al.*, 1985). The curve in Fig. 1 shows this fit transformed into a log–log axes. If plotted on *linear* axes, it would be a straight line with an x-axis intercept of −0.6 deg. It is clear that this curve (1) represents a reasonable fit to the data for eccentricities >0.5 deg, and (2) displays the characteristic Weber's law slope. We argued that in this large eccentricity regime thresholds appeared to represent a constant cortical distance. (3) At eccentricities <0.5 deg, this curve fails to predict the small bisection thresholds. We found that a second line, with an x-intercept near 0 was needed to fit the data at small eccentricities, as would be predicted on the basis of the filter hypothesis (Klein and Levi, 1985, 1987).

The present report focuses on the large eccentricity regime. We have pitted separation against eccentricity to test the spatial grain hypothesis directly. We used a 5-fold range of large separations, where it seemed unlikely that thresholds would be determined by the differential outputs of spatial filters. In this regime, the present results provide strong evidence *against* Weber's law. When separation is varied but eccentricity held constant, there is no Weber's law. Rather, the thresholds are approx. 0.2 deg (or about 2% of the 10 deg test line eccentricity) independent of separation. Our results also suggest that the eccentricity of the reference stimulus influences the precision of bisection judgements, so that the threshold is a constant fraction of the stimulus eccentricity plus an additive constant resulting from the eccentricity of the reference stimulus.

When we attempt to gauge the distance between widely separated objects it is unlikely that we do so on the basis of the outputs of large spatial filters; rather it appears that we make such judgements by estimating the cortical distance which separates the targets of interest. Our results suggest that the judgement of the separation of widely separated objects is similar to a distance measurement using a ruler on the cortex, in that the error of measurement is independent of the separation between objects.

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REFERENCES

Bedell H. E., Johnson M. H. and Barbeito R. (1985) Precision and accuracy of oculocentric direction for targets of different luminances. *Percept. Psychophys.* **38**, 135–140.

Burbeck C. A. (1987) Position and spatial frequency in large scale localization judgements. *Vision Res.* **27**, 417–428.

Cowey A. and Rolls E. T. (1974) Human cortical magnification factor and its relation to visual acuity. *Expl Brain Res.* **21**, 447–454.

Dow B. M., Snyder R. G., Vautin R. G. and Bauer R. (1981) Magnification factor and receptive field size in foveal striate cortex of the monkey. *Expl Brain Res.* **44**, 213–228.

Fechner G. T. (1858) Über ein Psychophysisches Grundgesetz. *Abh. Lpz Ges.* **VI**, 457–532.

Hering E. (1899) Über die Grenzen Der Sehscharfe. Ber. Math-Phys. Konigl. Sachs. Ges. Wiss. Lpz, *Naturw.* pp. 16–24.

Hirsch J. and Hylton R. (1982) Limits of spatial frequency discrimination as evidence of neural interpolation. *J. opt. Soc. Am.* **72**, 1367–1374.

Klein S. A. and Levi D. M. (1985) Hyperacuity thresholds of 1 sec: theoretical predictions and empirical validation. *J. opt. Soc. Am.* **A2**, 1170–1190.

Klein S. A. and Levi D. M. (1987) The position sense of the peripheral retina. *J. opt. Soc. Am.* **A4**, 1544–1553.

Levi D. M. and Klein S. A. (1983) Spatial localization in normal and amblyopic vision. *Vision Res.* **23**, 1005–1017.

Levi D. M., Klein S. A. and Aitsebaomo P. K. (1985) Vernier acuity, crowding and cortical magnification. *Vision Res.* **25**, 963–977.

Matin L. (1972) Eye movements and perceived visual direction. In *Handbook of Sensory Physiology*, Vol. VII/4, *Visual Psychophysics* (edited by Jameson D. and Hurvich L. M.). Springer, Berlin.

Toet A., van Eekhout H. L., Simons J. J. and Koenderink J. J. (1987) Scale invariant features of differential spatial displacement discrimination. *Vision Res.* **27**, 441–452.

Tootell R. B., Silverman M. S., Switkes E. and De Valois R. L. (1982) Deoxyglucose analysis of retinotopic organization in primate striate cortex. *Science, New York* **218**, 902–904.

Van Essen D. C., Newsome W. T. and Maunsell J. H. R. (1984) The visual field representation in striate cortex of the macaque monkey: asymmetries, anisotropies, and individual variability. *Vision Res.* **24**, 429–448.

Volkmann A. W. (1858) Über den Einfluss der Ilbung auf das Erkennen raumlicher Distanzen. *Leipzigger Ber.* **X**, 38–69.

Wertheim T. (1894) Über die Indirekte Schescharefe. *Z. Psychol. Physiol. Sinnesorg.* **7**, 172–189.

Westheimer G. (1975) Visual acuity and hyperacuity. *Invest. Ophthal.* **14**, 570–572.

Wilson H. R. (1986) Responses of spatial mechanisms can explain hyperacuity. *Vision Res.* **26**, 453–469.

Yap Y. L., Levi D. M. and Klein S. A. (1987a) Peripheral hyperacuity: 3-dot bisection scales to a single factor from 0 to 10 deg. *J. opt. Soc. Am.* **A4**, 1554–1561.

Yap Y. L., Levi D. M. and Klein, S. A. (1987b) Peripheral hyperacuity: isoeccentric bisection is better than radial bisection. *J. opt. Soc. Am.* **A4**, 1562–1567.