THE ROLE OF SEPARATION AND ECCENTRICITY IN ENCODING POSITION

DENNIS M. LEVI\textsuperscript{1} and STANLEY A. KLEIN\textsuperscript{2}

\textsuperscript{1}University of Houston, College of Optometry, Houston, TX 77004 and \textsuperscript{2}University of California, School of Optometry, Berkeley, CA 94720, U.S.A.

(Received 28 November 1988; in revised form 14 August 1989)

Abstract—We measured two- and three-line spatial interval and alignment thresholds for a wide range of separations and eccentricities. In order to avoid confounding the role of separation and eccentricity, the test lines were presented on iso-eccentric arcs, with radii between 0.625 and 10 deg. The iso-eccentric paradigm allows separation to be varied over a large range while holding eccentricity constant. Our main finding is that for all tasks, at all eccentricities, Weber's law fails at large separations. Our results are consistent with the hypothesis that distance judgements are limited by at least two factors: (1) when the separation of the iso-eccentric test lines is small with respect to the eccentricity position thresholds are proportional to the separation of the features, and show little dependence on eccentricity. This is Weber's law for position, and the threshold is approx. 0.03–0.05 times the stimulus separation; (2) when the separation of the iso-eccentric test lines is comparable in size to the eccentricity, position thresholds are proportional to the target eccentricity, and are essentially independent of separation. In this "eccentricity regime", position discrimination thresholds are \( k \frac{\text{ECC}}{S} \) where ECC is the stimulus eccentricity in degrees, and \( k \) is a fraction of the eccentricity (\( \approx 0.01-0.03 \)). The observer uses whichever mechanism is more sensitive to the stimulus. Under the conditions of our experiments, the observers perform a Weber computation when the separation, \( S \), between the pair of iso-eccentric test lines is less than about 0.5 \( \times \) ECC, and apply a "cortical rule" when \( S \) is greater than about 0.5 \( \times \) ECC. If the angle subtended by the fixation point and the pair of iso-eccentric test lines is considered, then for both the two-line and the three-line tasks, when the angle is less than approx. 30 deg, thresholds are proportional to separation, and when it is between 30 and 180 deg, thresholds are proportional to eccentricity.

Weber's law
Psychophysica
Periphery
Spatial interval
Alignment
Position discrimination
Hyperacuity
Localization

INTRODUCTION

The German physiologist E. H. Weber discovered that two relatively heavy weights must differ by a greater amount than two relatively light weights, in order to be perceived as different. More precisely, Weber found that the size of the difference threshold was a linear function of the stimulus weight. From a large number of discrimination experiments in several modalities, Weber derived a pervasive description of sensory perception, known to us as Weber's law—namely, that the just-noticeable difference in stimulus intensity is a constant fraction of the stimulus (Weber, 1834). Some of the earliest evidence for Weber's law came from studies of position judgements (Volkmann, 1863; Fechner, 1860). While positional acuity is often thought of in terms of the very precise spatial thresholds which are obtained when the target features are close (Westheimer, 1975; Westheimer & McKee, 1977a; Klein & Levi, 1985), one of the most striking features of positional acuity is that over a wide range of conditions, the position threshold is approximately proportional to the separation of the reference features. Weber's Law is ubiquitous in spatial vision. For example bisection (Volkmann, 1863; Fechner, 1860; Levi & Klein, 1983; Klein & Levi, 1983, 1987), Vernier (Sullivan, Oatley & Sutherland, 1972; Andrews, Butler & Buckley, 1973; Beck & Schwartz, 1979; Beck & Halloran, 1985) and separation discrimination experiments (Westheimer & McKee, 1977a; Andrews & Miller, 1978; Hirsch & Hylton, 1982; Yap, Levi & Klein, 1989) all reveal the characteristic proportionality between feature separation and threshold. Thus, an understanding of Weber's law is important to our understanding of space and distance perception.

This paper explores the idea that at large separations Weber's law might be based upon the spatial sampling grain of the visual system (Hering, 1899; Matin, 1972; Burbeck, 1987;
The notion here is that the spatial grain of the cortex (\( M^{-1} \) which is the number of degrees of visual angle per mm of cortex) varies linearly with eccentricity; thus the position of the features becomes increasingly uncertain as their separation, and hence their eccentricity increases.

There is experimental support to suggest that both the separation and the eccentricity of the stimulus features may be important in limiting the precision of spatial localization. Klein and Levi (1987) measured position acuity using a spatial interval discrimination task. In this experiment, a foveal reference line appeared, and was followed by a pair of briefly flashed peripheral "test" lines. The separation between the test and reference lines varied from 0.05 to 10 deg. Because of a cusp in the data, a two-line fit was required. For separations/eccentricities up to about 20 min. the thresholds increased very rapidly with separation, consistent with the filter hypothesis. For larger separations/eccentricities (greater than about 20' with a foveal reference) the position thresholds appear to vary in a manner consistent with recent estimates of the spatial sampling grain of the cortex (Dow, Snyder, Vautin & Bauer, 1981; Tootell, Silverman, Switkes & De Valois, 1982; Van Essen, Newsome & Maunsell, 1984).

A second line of experimental evidence for the role of eccentricity is from experiments in which separation was pitted against eccentricity by measuring spatial interval discrimination thresholds for short lines presented on an iso-eccentric arc with a radius of 10 deg, so that the interline separation could be varied while holding the stimulus eccentricity constant (Levi, Klein & Yap, 1988). In those experiments we used a five-fold range of large separations (from 2 to 10 deg), where it seemed unlikely that thresholds would be determined by the differential outputs of spatial filters. In this regime, the results provided strong evidence against Weber's law. When separation was varied but eccentricity held constant, there was no Weber's law. Rather, the thresholds were approx. 0.16 deg (or about 0.016 times the 10 deg test line eccentricity) independent of separation. Figure 1 summarizes some of the data for observer DL from Levi et al. (1988) in which two (triangles) or three lines (filled squares) were flashed simultaneously over an expanded range of separations (from approx. 0.3 to 10 deg). Also shown are new data of DL, obtained at half the viewing distance (circles; the implications of this control experiment will be discussed in the Discussion section). Note that at separations greater than about 2 deg, thresholds are approximately constant, suggesting that separation is not the limiting factor. We suggested that for large separations, the stimulus eccentricity limits performance. In contrast, for separations < about 2 deg, thresholds are proportional to separation. Taken together, these studies suggest that both feature separation and eccentricity may serve to limit the precision of spatial judgements.

Weber's law is so ubiquitous in sensory psychology that the strong violation of Weber's law evident in our iso-eccentric data (Levi et al., 1988) is quite surprising; thus the present paper examines the generality of our iso-eccentric results by testing several new observers on two-line and three-line spatial interval and alignment thresholds on iso-eccentric arcs of different radii, from 0.625 to 10 deg. Our main finding is that Weber's law for position fails when the
separation is large with respect to the eccentricity. We consider several models for position acuity, and reject both Weber's law, and a square root law for position discrimination at large separations. Rather, our results are compatible with a model in which two processes limit position judgements.

METHODS

Three-line experiments

The stimuli and methods were essentially identical to those described in Levi et al. (1988). Briefly, the stimuli consisted of three short, bright lines (≈15 times the line detection threshold) generated on the dark screen of a computer (Commodore Pet). The room lights were extinguished. In the main three-line experiments, for a target eccentricity of 10 deg, the screen subtended 28 \times 17 deg, and was halved for each doubling of eccentricity. For the spatial interval discrimination task, the lines were vertical. A “reference” line was presented for 1 sec. A pair of flanking “test” lines were then presented in one of the five randomly chosen positions (equidistant from the reference line, or displaced 1 or 2 units to the left or right) and 200 msec later all three lines were extinguished. The three-line spatial interval task is illustrated in Fig. 2 (top left) for both a large and a small separation (the separation “SEP” is indicated in the figure by the arrows). For the widest separation (separation equal to the eccentricity) the reference line also serves as a fixation point. The long duration of the reference line makes it a good stimulus for fixation, and since the test lines appear on either side of the reference line, anticipatory eye-movements to fixate the test lines are unlikely (and unhelpful). For all other separations, the observer fixated a long thin horizontal line above the target lines (and orthogonal to their orientation). Jittering the absolute position of the stimuli with respect to the fixation point, eliminated it as a useful compari-

Fig. 2. Schematics of the three-line (left) and two-line (right) stimuli used for spatial interval discrimination (top) and alignment (bottom) for both large and small separations. Note that the three-line stimuli are identical, except for the presence of a reference line midway between the test lines. The F denotes foveal fixation. The stimuli illustrated here are presented on an iso-eccentric arc in the lower visual field; however, in several two-line experiments, the stimuli were presented in the right field (this can be envisaged by rotating the figure 90 deg clockwise). The strategy of presenting the test targets on an iso-eccentric arc allows separation to vary over a large range, while keeping eccentricity constant.
son target. The observer's task was to judge the direction and magnitude of the horizontal displacement of the test lines. This is a spatial interval judgement, since shifting the test lines to the right will make the interval between the middle line and the right test line larger than the interval between the middle line and the left test line. This task is sometimes called a bisection task (Volkmann, 1863; Klein & Levi, 1985, 1987). The point that we wish to emphasize, is that in our task (in which the outer lines are displaced) as well as in the more conventional bisection task (in which the center line is displaced), performance will be limited by the precision with which the more peripheral (outer lines) can be localized. Following the observer's rating scale response the computer provided feedback as to both the direction and magnitude of the offset. In some experiments we examined the effect of stimulus onset and duration, by presenting all three lines simultaneously for 200 msec or for 1 sec.

For the alignment task, the lines were horizontal, and the observer's task was to make an alignment judgement about the vertical displacement of the peripheral test lines with respect to the middle reference line, and about the magnitude of the displacement. The three-line alignment task is illustrated in Fig. 2 (bottom left) for both a large and a small separation (the separation is indicated in the figure by the arrows). In some experiments, the test lines were presented for 1 sec.

The edges of the screen could, in principle, provide a reference which could influence the results. In the present experiments, edge effects were minimized by three strategies. First, for both interval and alignment tasks, the position of the entire display was randomly jittered with respect to the fixation target and the screen edges, by approx. 20% of the target eccentricity (i.e. the jitter was considerably larger than the threshold). Secondly, halving the viewing distance (so that the screen size at an eccentricity of 10 deg was 56 x 34 deg) had no effect upon the thresholds (note that the critical screen dimension, the width, was 56 deg). In this condition, and in all of the two-line experiments, the field size is sufficiently large that the edges are quite distant (i.e. the field size is almost six times larger than the eccentricity being tested). Thirdly, the visibility of the edges was low, since the target lines were presented on a dark background, and the room lights were extinguished.

One way to avoid confounding the role(s) of separation and eccentricity, is to present the test lines on iso-eccentric arcs. The arc radii varied between 0.625 and 10 deg and were presented in the lower visual field. For each iso-eccentric arc, the stimuli were identical, and variations in eccentricity were achieved by varying the viewing distance. For most of the three-line experiments, the lines were approx. 30' long by 3' wide for the 10 deg arc at the viewing distance of 39 cm (as in Levi et al., 1988). Each halving of the eccentricity was accomplished by doubling the viewing distance. At the smallest eccentricity (0.625) the lines were approx. 2' by 0.2' at the viewing distance of 6.24 m. This strategy ensures that the lines were well localized at the nominal test eccentricity. In control experiments and in all the two-line experiments, the viewing distance was halved, so that for the 10 deg arc, the lines were 1 deg long by 6' wide, and for the 1.25 deg arc (the smallest tested in the two-line experiments), the lines were 8' by 0.8'.

In order to obtain a criterion-free measure of the threshold for correct discrimination of the position of the test lines, d' values for each stimulus were calculated using a maximum-likelihood fit to the rating scale data (Dorfman & Alf, 1969), and interpolation to a \( d' = 0.675 \) (equivalent to 75% correct in a yes-no task such as that used by Westheimer & McKee, 1977a) was used as a measure of the discrimination threshold. This is equivalent to a multiple-criterion probit analysis. Runs which were too difficult or too easy (i.e. in which the d' values for the two-unit offset were less than 0.8 or greater than 3.8) were excluded from analysis, resulting in a rejection rate of about 5%. All other runs were included in the analysis, and the threshold values reported here are the geometric means of 2–6 runs per condition weighted by the inverse variance. The error bars include both within and between session variability (Klein & Levi, 1987).

Two-line experiments

In the main experiments we used the three-line spatial interval and alignment tasks, because they provide an explicit and stable reference, thus minimizing the role of memory and cognitive factors. However, in this stimulus arrangement, the reference point is not on the iso-eccentric arc so its positional uncertainty will differ from the uncertainty of the test lines. In order to assess what effects this might have, we also conducted two-line spatial interval dis-
discrimination and two-line alignment experiments, in which both lines were flashed simultaneously for 200 msec, and both were presented on the iso-eccentric arc. Figure 2 shows examples of both the three-line (left) and two-line (right) stimuli used for spatial interval discrimination (top) and alignment (bottom). In each panel are two stimuli representing large and small separations. Note that the two-line and three-line stimuli are similar in most respects, however, they differ with respect to the presence of a reference line midway between the test lines, and the relevant separation is doubled in the two-line task (see Fig. 2). There are several other differences between the two-line and three-line experiments: (1) in the two-line experiments, the viewing distance was halved, so that the screen was $56 \times 34$ deg at an eccentricity of 10 deg, and the test lines were longer and wider (similar to the half distance control three-line experiment). Thus, in the two-line experiments the field size was about three times the largest separation; (2) the two-line experiments require a remembered reference, whereas the three-line task has an explicit reference. If the observer actually compares the two intervals in the three-line case, then thresholds should be a factor of two better for three-line than for two-line experiments. In the Appendix, we consider the possible effects of the reference, and the role of memory in detail; (3) for the largest separation in the two-line experiments, where the test lines are presented on either side of fixation, no fixation point was used, whereas for the three-line experiments, at the widest separation, the reference line served as a fixation point. For all other separations, the same long fixation line, oriented orthogonal to the test lines served as a fixation target. As in the three-line experiments, jittering the absolute position of the test lines renders the fixation lines and the screen edges uninformative. All other stimulus and experimental conditions were identical to those of the three-line experiments. In the experiments illustrated in Fig. 2, the iso-eccentric arcs were presented in the lower visual field, however, in several two-line experiments, the stimuli were presented in the right field (this can be envisaged by rotating the figure). In the two-line spatial interval discrimination experiments, the observer’s task was to judge the separation with respect to an internal reference (Westheimer & McKee, 1977b; Levi et al., 1988); for the alignment task, the observer judged whether the left line was higher or lower than the right line. Note that for the three-line tasks, separation is measured from the reference line to the test line. For the two-line task, separation is measured between the two test lines (Fig. 2) because that is the separation with the test cue. As will be shown in the Results section, thresholds are higher in the two-line task than in the three-line task, however, the effects of separation and eccentricity are essentially identical. The Appendix examines alternative decision strategies for threshold that are relevant for comparing two-line and three-line tasks and the effect of uncertainty in the memorized reference (for the two-line task) vs the uncertainty in the location of the middle reference line (for the three-line task).

**Observers**

Four observers with normal or corrected-to-normal vision participated in these experiments; DL, an author, and three undergraduate students who were given extensive practice, but who were naive as to the purpose of the experiments.

**RESULTS AND DISCUSSION**

**Three-line spatial interval**

Figures 3, 4 and 5 summarize the spatial interval data of observers JT, KH and CN at a series of iso-eccentric loci. Plotted here are the position discrimination thresholds (in deg) against the separation of the test and reference lines (in deg) on log–log axes. Each symbol represents a different iso-eccentric arc, whose radius is noted on the right-hand side of each curve. At each eccentricity, the rightmost datum (i.e. the largest separation) represents the case in which the reference line is presented to the fovea (see Fig. 2; this point is considered further in the section on “Thresholds with a foveal reference line”). The dashed line in each figure represents Weber’s law for position such that threshold, Th, is given by $Th = SEP \cdot k_{sep}$ (i.e. threshold is a constant fraction $k_{sep}$, of separation). The value of $k_{sep}$ for each of the figures is shown in Table 2. Each horizontal solid line segment represents the spatial grain hypothesis, with the line at $(ECC + E_2) \cdot k_{ecc}$, where ECC is the eccentricity of the test lines (i.e. the radius of the iso-eccentric arc), $k_{ecc}$ is a fraction of the eccentricity and is shown in Table 2, and $E_2$ is a constant, representing the rate of variation with eccentricity. $E_2$ is small, and to simplify the modeling, it will be ignored for the present (it will be discussed further in the section on
ISO-ECCENTRIC SPATIAL INTERVAL SUMMARY

Fig. 3. Three-line spatial interval data of observer JT at a series of iso-eccentric loci. Plotted here are the position discrimination thresholds (in degrees) against the separation of the test and reference lines (in degrees) on log–log axes. Each symbol represents a different iso-eccentric arc, whose radius is noted on the right-hand side of each curve. The dashed line represents Weber’s law for position—i.e. threshold, \( \text{Th} = k \times \text{SEP} \). The Weber fraction is given in Table 2, in the column labelled \( k_w \) for each figure. Each horizontal solid line segment represents the spatial grain hypothesis, with the line at \( \text{Th} = (E + E_2) \times k \) where \( E \) is the eccentricity of the test lines (i.e. the radius of the iso-eccentric arc), \( k \) is a fraction of the eccentricity (the value of \( k \) is given in Table 2 in the column labelled \( k_w \) for each figure) and \( E_2 \) is a constant. At each eccentricity, the rightmost datum (i.e. the largest separation) represents the case in which the reference line is presented to the fovea (see Fig. 2). Note that for each eccentricity, at large separations, there is an apparent flattening of the curves, so that Weber’s law does not obtain.

"Position threshold vs eccentricity”). The lines fit to the data will be discussed further in the section on “Modeling the effects of separation and eccentricity”. Consider the data of observer JT (Fig. 3) at 10 deg. As the separation decreases, there is a four to five-fold range of separations, over which thresholds are more or less constant. Thus, at ECC = 10 deg, thresholds are more or less constant for separations from about 2–10 deg. This regime, which we shall refer to as the “eccentricity regime” corresponds to the range of separations which we have previously reported on (Levi et al., 1988). Note that this represents a severe failure of Weber’s law, since according to Weber’s law, thresholds should increase by a factor of five over this range of separations. At smaller separations, thresholds first decrease more or less along the Weber’s law line (i.e. the “Weber’s law regime”), and then tend to increase again. The increased threshold at the smallest separation (the leftmost point) represents an effect of “crowding”, which has been noted previously (Yap, Levi & Klein, 1987a). At smaller eccentricities, the entire function relating position threshold to separation is shifted to the left (i.e. smaller separations) and down (lower thresholds). While there is some variation in the pattern of results across eccentricities, it is clear that at each eccentricity there is a flattening of the data at large separations, so that: (1) at large separations thresholds are not proportional to separation; and (2) at large separations, thresholds appear to increase with increasing
Role of separation and eccentricity in encoding position

ISO-ECCENTRIC SPATIAL INTERVAL SUMMARY

Weber's law

Fig. 4. Three-line spatial interval data for observer KH at iso-eccentric arcs from 1.25 to 10 deg. All the details are identical to those of Fig. 3. The arrows at the smallest separations for which we attempted to measure thresholds indicate that observer KH was unable to make spatial interval judgements at these small separations. Despite many attempts, her thresholds were unmeasurably large.

eccentricity. We also used a 1 sec viewing duration to measure three-line spatial interval discrimination thresholds for JT on a 10 deg iso-eccentric arc and found similar results (shown in Tables 1 and 2).

Figures 4 and 5 show spatial interval discrimination data for observers KH and CN for iso-eccentric arcs from 1.25 to 10 deg. Their results are similar to those of JT, in showing a failure of Weber’s law at large separations. It is also clear that for each observer and eccentricity, at small separations, thresholds decline as separation declines, and then increase markedly at the smallest separation, as a consequence of “crowding”. However, strong individual differences are worth noting. First, neither of these observers (nor DL who is very well practiced in peripheral position judgements) demonstrate quite as acute positional acuity as JT at large separations (i.e. in the eccentricity regime). For example, in the eccentricity regime KH and CN’s spatial interval discrimination thresholds are about 0.016 eccentricity, compared to 0.01 eccentricity for JT. In our previous study using an iso-eccentric arc with a radius of 10 deg, we noted that as the line separation increased from 2 to 10 deg, thresholds actually declined slightly (Levi et al., 1988). We attributed this to a possible effect of the eccentricity of the reference line midway between the two test lines. There was considerable intersubject variability in the degree to which thresholds declined. This can also be seen in the present study. KH shows a marked decrease in thresholds in the eccentricity regime at the three largest eccentricities, while CN and JT show either a slight or no decline in thresholds. The observers also differ to some extent at the smallest separations. For example, CN shows marked increases in threshold at each eccentricity at the smallest separations. Despite many attempts, KH was unable to obtain mea-
surable thresholds at the smallest separations at any eccentricity (as indicated by the arrows).

Three-line alignment

To test the generality of our results, we have also measured thresholds for alignment (similar

Table 1. Non-linear analysis of iso-eccentric data: reduced chi-squared values

<table>
<thead>
<tr>
<th>Task</th>
<th>Observer</th>
<th>Data from figure</th>
<th>Weber’s law</th>
<th>Fit functions</th>
<th>Variable slope</th>
<th>Two-mechanism model</th>
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<tbody>
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<td>3-line interval</td>
<td>DL</td>
<td>1</td>
<td>36.9 ± 2.4</td>
<td>7.9 ± 1.1</td>
<td>1.2 ± 0.46</td>
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<td></td>
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<td>3</td>
<td>29.3 ± 1.2</td>
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<tr>
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<td>KH</td>
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<td>31.9 ± 1.4</td>
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<td>CN</td>
<td>5</td>
<td>26.6 ± 1.3</td>
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<td>1 sec duration</td>
<td>JT</td>
<td>Not shown</td>
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<td>16 ± 2.1</td>
<td>3.9 ± 1.0</td>
<td>3.7 ± 1.1</td>
<td>0.9 ± 0.54</td>
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*The standard error SE of the reduced chi-square ($\chi^2_r$) is obtained from the formula $SE = (3\chi^2_r/df)^{1/4}$ where df = the number of degrees of freedom.

*The mean represents the arithmetic mean, weighted by the inverse variance.

*Figure 13 includes 12 of the 18 data points from Fig. 9B, so it is interesting to note that the fits are quite similar.
to Vernier thresholds). There is a second motivation for choosing the alignment task. Alignment thresholds are widely thought to represent an orientation judgement (Sullivan et al., 1972; Andrews et al., 1973; Watt, 1984). For example, the well-documented proportionality between the alignment threshold, and the feature separation (i.e. Weber's law behavior) would be a direct consequence of a constant angular tilt cue (see Sullivan et al., 1972). The generality

### ISO-ECCENTRIC ALIGNMENT SUMMARY

![Graph showing Weber's law](image)

**Fig. 6.** Three-line alignment data of JT at iso-eccentric arcs from 0.625 to 10 deg. The results are qualitatively similar to those for spatial interval discrimination, in that Weber's law appears to fail at large separations.

<table>
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<th>Two-mechanism model fit</th>
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<td>k&lt;sub&gt;m&lt;/sub&gt;</td>
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<td></td>
<td>CN</td>
<td>5</td>
<td>0.22 ± 0.04</td>
<td>0.08 ± 0.003</td>
</tr>
<tr>
<td>(1 sec duration)</td>
<td>JT</td>
<td>Not shown</td>
<td>0.30 ± 0.06</td>
<td>0.051 ± 0.004</td>
</tr>
<tr>
<td>2-1 interval</td>
<td>DL</td>
<td>8</td>
<td>0.51 ± 0.03</td>
<td>0.061 ± 0.003</td>
</tr>
<tr>
<td></td>
<td>DL</td>
<td>10A</td>
<td>0.50 ± 0.05</td>
<td>0.10 ± 0.01</td>
</tr>
<tr>
<td></td>
<td>DL</td>
<td>13</td>
<td>0.47 ± 0.07</td>
<td>0.07 ± 0.003</td>
</tr>
<tr>
<td></td>
<td>JT</td>
<td>9A</td>
<td>0.48 ± 0.05</td>
<td>0.06 ± 0.002</td>
</tr>
<tr>
<td></td>
<td>CN</td>
<td>9B</td>
<td>0.48 ± 0.04</td>
<td>0.068 ± 0.003</td>
</tr>
<tr>
<td></td>
<td>CN</td>
<td>13</td>
<td>0.51 ± 0.05</td>
<td>0.068 ± 0.003</td>
</tr>
<tr>
<td>3-1 alignment</td>
<td>JT</td>
<td>6</td>
<td>0.22 ± 0.03</td>
<td>0.031 ± 0.001</td>
</tr>
<tr>
<td></td>
<td>KH</td>
<td>7</td>
<td>0.33 ± 0.03</td>
<td>0.031 ± 0.001</td>
</tr>
<tr>
<td>2-1 alignment</td>
<td>DL</td>
<td>10B</td>
<td>0.6 ± 0.06</td>
<td>0.039 ± 0.003</td>
</tr>
</tbody>
</table>
of the role of orientation has been questioned by several investigators (e.g. Westheimer & McKee, 1977a; Watt, Morgan & Ward, 1983; Watt, 1984) who have suggested that there may also be spatial limitations to Vernier acuity. Thus, the iso-eccentric paradigm may prove useful in distinguishing between these hypotheses.

Figures 6 and 7 show the three-line alignment data of JT and KH at iso-eccentric arcs from 0.625 to 10 deg. The results are qualitatively similar to those for spatial interval discrimination in showing a failure of Weber's law at large separations. At each eccentricity, there is a strong flattening of the curves at large separations, so that Weber's law does not obtain. Note that KH's iso-eccentric curves at the largest (10 deg) and at the smallest eccentricities (0.625 deg) were obtained with 1 sec durations. The data are qualitatively similar to the short duration data at each of the other eccentricities tested (0.2 sec duration), and quantitatively similar to the short duration data of JT at the same eccentricities. These results show that the alignment task cannot be explained solely on the basis of the orientation cue. A constant orientation limitation would have resulted in a Weber's law behavior.

Two-line spatial interval discrimination and two-line alignment

To further examine the generality of our results, and to ensure that the failure of Weber's law at large separations is not a consequence of the central reference line (which only falls near the iso-eccentric arc at the small separations), we also conducted several two-line interval discrimination experiments, in which there was no reference line (see Fig. 2, right). Figure 8 shows data of DL for two-line spatial interval discrimination for a series of iso-eccentric arcs with radii from 1.25 to 10 deg in the lower field. These results, like the three-line data, show that for each eccentricity there is a failure of Weber's law (flattening) at large eccentricities. It is also interesting to note that the two-line data do not

ISO-ECCENTRIC ALIGNMENT SUMMARY

Fig. 7. Three-line alignment data of KH at iso-eccentric arcs from 0.625 to 10 deg. The data are similar to those of JT in Fig. 6. The data at eccentricities of 0.625 and 10 deg were obtained with a 1 sec test line duration, whereas all the other data were with 0.2 sec.
reveal the strong crowding effects evident in the three-line data at small separations, so that at each eccentricity, there is a large range of small separations where thresholds appear to be more or less proportional to separation, falling close to the Weber's law line. A limited set of two-line interval data is also shown for observers JT and CN in Fig. 9 (and will be discussed further below). Note that separation is defined differently for the two- than for the three-line task* (see Fig. 2), so that in the two-line task, the largest separation on any iso-eccentric arc is equal to twice the eccentricity. Consider the angle between the fixation point, and the pair of test lines on an iso-eccentric arc with a radius of 10 deg. When the angle is 180 deg the separation is defined as 10 deg for the three-line task and 20 deg for the two-line task. These two-line data illustrate clearly that Weber's law fails at large separations even when there is no central reference line, as in our three-line experiments.

Figure 10 shows two-line spatial interval (Fig. 10A) and two-line alignment (Fig. 10B) thresholds for observer DL obtained on an iso-eccentric arc with a radius of 5 deg in the right field. These results, like the three-line data, also show the failure of Weber's law at large separations. For both tasks, thresholds are more or less proportional to separation at small separations, and show a strong departure from Weber's law at large separations. These data obtained on an iso-eccentric arc in the right visual field, also serve to show that the failure of Weber's law at large separations is not a consequence of making comparisons across the vertical midline.

Scaling the results with eccentricity

If target eccentricity is a critical limiting factor in precise position judgements then the thresholds obtained on arcs of different radii

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*For both the two-line and three-line experiments, the separation is defined as the interval containing the cue.
Fig. 9. Two-line spatial interval discrimination data of JT for a series of iso-eccentric arcs with radii from 1.25, 2.5 and 5 deg in the lower field.

should scale simply with eccentricity. Figure 11 shows that this is indeed the case. Here the three-line spatial interval data of CN (Fig. 5), the three-line alignment data of JT (Fig. 6) and the two-line spatial interval data of DL (Fig. 8) have been replotted in Figs 11A, B and C, respectively. The abscissa in each of these figures represents the angle \( \theta \) between the
Role of separation and eccentricity in encoding position

Fig. 10. Two-line spatial interval (A) and two-line alignment (B) thresholds for observer DL obtained on an iso-eccentric arc with a radius of 5 deg in the right field.

fixation point (F in Fig. 2) and the pair of iso-eccentric test lines. Plotting $\theta$ has the effect of taking into account changes in scale with eccentricity, since the same angle necessitates larger separations as eccentricity increases. Note that the angle provides only an approximate scaling because it does not take into account the non-zero $X$-axis intercept found when threshold is plotted against eccentricity. Since, the $X$-axis intercept of our data was on average about $-0.45$ deg eccentricity, its effects are negligible except at very small eccentricities. The angle also facilitates direct comparison between the two-line and three-line data. The ordinate in each figure is the position threshold specified as a fraction of ECC (the target eccentricity). Note that the effect of this transformation is to collapse the widely disparate data from different eccentricities (about a 10-fold range in thresholds at large separations from 1.25 to 10 deg eccentricity) into a more or less unitary function for each of the three tasks. It is also
interesting to note that the transformation of the abscissa provides a reasonable (although not perfect) horizontal transformation of the curves, so that at each eccentricity, for $\theta$ less than about 30–40 deg, thresholds are proportional to separation, while for $\theta$ larger than about 30–40 deg, they are proportional to eccentricity.

**Modeling the effects of separation and eccentricity**

In this section, we quantitatively test several models for iso-eccentric spatial position discriminations. Specifically, we used nonlinear regression analysis to fit the threshold data from each eccentricity according to the following models: (1) Weber's law ($\text{Th} = k \cdot \text{SEP}$), i.e. a straight line with unity slope (log–log); (2) square root law ($\text{Th} = k \cdot \text{SEP}^{0.5}$), i.e. a straight line with a slope of 0.5 (log–log). This behavior would be expected if component intervals, each with an independent error were cumulated; (3) a straight line whose slope (log–log) is varied for best fit ($\text{Th} = k \cdot \text{SEP}^p$); (4) a two-mechanism model, i.e. a broken line with two parameters, $k_1$ and $k_2$ such that thresholds are a fraction of separation at small separations, and a fraction of eccentricity at large separations. See Fig. 1 for an example of the two-mechanism fit. Note that in fitting each of the 46 iso-eccentric curves (representing well over 100,000 trials) data at very small separations where crowding occurs (less than 0.08 $\times$ ECC) were omitted. The results are presented in Table 1.

The first three columns in Table 1 specify the task, the observer and the figure from which the data were obtained. Columns 4–7 show the goodness-of-fit of each model, by reporting the reduced chi-square values (chi-square/degrees of freedom) $\chi^2$, and their error bars for each of the model fits. In fits 1 and 2 (Weber's law and slope fixed at 0.5) the number of parameters in the fit Fig. 11. The three-line spatial interval data of CN (Fig. 5), three-line alignment data of JT (Fig. 6) and the two-line spatial interval data of DL (Fig. 8) are replotted here with the abscissa representing the angle, $\theta$, between the fixation point (F in Fig. 2), and the pair of iso-eccentric lines. This takes into account eccentricity, since the same angle necessitates larger separations as eccentricity increases. The ordinate is the position threshold specified as a fraction of $E$ (the target eccentricity) + $E_2$. The effect of this transformation is to collapse the widely disparate data from different eccentricities into a more or less unitary function. At each eccentricity, for angles $\theta$ less than about 30–40 deg, thresholds are proportional to separation, while for angles larger than 40 deg, they are proportional to eccentricity.
equals the number of eccentricities since there is one parameter for each intercept. For fits 3 and 4 (variable slope and two-mechanism model) there are twice as many free parameters. \( \chi^2 \) takes this difference into account since the number of degrees of freedom is equal to the number of data minus the number of parameters. Note that the smaller \( \chi^2 \) the better the fit to the data. For a "perfect" fit in which the chi-square is solely due to uncorrelated noise, \( \chi^2 \) should be approximately unity.

Note that in every instance, the unity slope line, i.e. Weber's law, provides a very poor fit to the data, since the mean \( \chi^2 \) is approx. 28 (range 14.1–59.4). Both the square root model (slope of 0.5) and the variable slope (below we show that the best fitting fixed slope is about 0.3) provide a better fit to the data than do Weber's law, with mean \( \chi^2 \) of about six and four, respectively. However, it is clear from Table 1 that the two-mechanism model provides the best fit to the data set taken as a whole, with a mean \( \chi^2 \) of less than two. For two-line and three-line interval discrimination, and for two-line alignment, the two-mechanism model provided a significantly better fit than any other model at the 0.01 level or better. The two-mechanism model also provided a significantly better fit than Weber's law or the square root law to the three-line alignment data; however, for three-line alignment, \( \chi^2 \) was not significantly different for the variable slope and two-mechanism models even at the 0.05 level. In Table 2 (discussed in detail below), it can be seen that the best fitting variable slopes for three-line alignment were extremely flat, approx. 0.25. We know of no theory that predicts such a function. Rather, we believe that this fit represents a compromise between a near zero-sloped eccentricity regime, and a very small region where threshold depends upon separation, but possibly with a slope less than unity (inspection of Figs 6 and 7 suggest that Weber's law may not obtain even at small separations). Because this separation dependent region is small (constrained by crowding at small separations, and by the eccentricity regime at large separations), and the data is somewhat sparse, a single almost flat line provides a slightly better fit than the two-mechanism model. Note that the two-mechanism model provides a better fit to every data set than does either Weber's law or a square root model.

Table 2 presents the parameters obtained by fitting: (1) the variable slope model; and (2) the two-mechanism model to the entire data set from each figure listed in column 3. The number of parameters in the variable slope model is the number of eccentricities (one parameter for each intercept) plus one parameter for the slope. Column 4 shows the best fitting slope. The two-mechanism model has three parameters, \( k_{sep} \), the Weber fraction for the separation region (\( Th = k \cdot \text{SEP} \)) and \( k_{ecc} \), the eccentricity fraction (\( Th = k \cdot \text{ECC} \)), and \( E_2 \) (discussed below). Columns 5 and 6 give the values of \( k_{sep} \) and \( k_{ecc} \). There are several points worth noting in Table 2. First, the best fitting straight lines tend to have a rather shallow slope (mean slope \( \approx 0.3 \)). It is interesting to note that for the two-line spatial interval data, the best fitting straight line had a slope close to 0.5. This is probably due to the fact that the separation and eccentricity regimes are about equal in length, and the results of averaging two segments, one with a slope of about 1, and the other with a slope of 0, will be a slope of about 0.5. The important point here is that the two-mechanism model provides a better fit to the data (lower \( \chi^2 \) in Table 1) than does a single straight line. The parameters of the two-mechanism model fit are also of some interest. In the Weber's law regime \( k_{sep} \) (the constant fraction of separation) is on average about 0.05; however, it is about a factor of two better for three-line alignment than for three-line spatial interval discrimination. Similarly, in the eccentricity regime, \( k_{ecc} \) (the constant fraction of eccentricity) varies according to the task. For example, for three-line alignment it is about 0.008, for three-line interval discrimination it is approx. 0.015, while for two-line interval discrimination, it is about a factor of two worse. The difference between two-line and three-line thresholds in the eccentricity regime is examined further in the section on "two-line vs three-line thresholds" below, and in the Appendix.

Our results suggest that in a variety of iso-eccentric spatial position discrimination tasks Weber's law fails. Moreover, the bulk of the data is not well fit by a square root law model, in which the visual system cumulates component intervals, each with its own independent errors or by a single straight line function. Rather, the results, taken as a whole, are consistent with the hypothesis that two mechanisms operate to limit position judgements: (1) at small separations (but larger than the region of crowding), thresholds are proportional to separation, and show only a small dependence upon
eccentricity; and (2) at large separations, thresholds are proportional to eccentricity and depend little upon separation. Note that the lines fit to the data in all of the figures, representing both the Weber's law regime and the eccentricity regime, represent the parameters obtained from the two-mechanism model fit shown in Table 2.

The transition from Weber's law to the eccentricity regime

Our results thus far are consistent with the suggestion that two mechanisms limit position judgements: (1) a Weber computation approx. \( \text{Th} = k_* \cdot \text{SEP} \); and (2) a "cortical ruler" with a precision of about \( \text{Th} = k_* \cdot \text{ECC} \) (where ECC is the target eccentricity). The observer uses whichever mechanism is more precise. The Weber computation is more sensitive when the target separation is small with respect to the eccentricity, while the cortical ruler is more sensitive at separations which are comparable to the target eccentricity. Setting the thresholds equal at the breakpoint gives: \( k_* \cdot \text{SEP} = k_* \cdot \text{ECC} \) or \( k_*/k_* = \text{SEP}/\text{ECC} = \sin \theta \). Thus, the ratio of the two Weber fractions gives the angle \( \theta \) for the two-line task (or \( \theta/2 \) for the three-line task). The angle \( \theta \) at which the transition occurred is shown in the last column of Table 2. Theta was determined from the two-mechanism fits to each data set (fits at each eccentricity give very similar points of transition). Interestingly, the point of transition was, on average, at an angle of about 25 deg, and the largest transition was at about 45 deg. This is equivalent to a separation \( S \) between the pair of iso-eccentric test lines of between about 0.4 and 0.6 times the target eccentricity. These points of intersection represent a crude estimate of the largest separation at which the Weber's law computation provides the more reliable position signal. The data suggest that the range of separations over which Weber's law provides the most sensitive estimate of position is quite small when considered in terms of the angle. This point is illustrated schematically in Fig. 12. The shaded region encompasses an angle (\( \theta \)) of approx. 30 deg. For pairs of iso-eccentric test lines which lie in the unshaded region (\( \theta \) greater than about 30 deg), thresholds are proportional to the eccentricity of the lines. For test lines lying within the lightly shaded region (\( \theta \) less than about 30 deg), thresholds are proportional to the separation of the lines. This formulation holds for both the two-line and three-line tasks. For test lines lying in the darkly shaded region, crowding effects limit three-line (but not two-line) spatial interval discrimination, and performance is degraded (Yap et al., 1989).

In order to further test the notion that at large separations (angles greater than about 40 deg) thresholds are independent of separation, we measured two-line spatial interval discrimination thresholds under a new condition: we paired two large separations, an angle of 50 deg, and an angle of 120 deg at eccentricities of 1.25, 2.5 and 5 deg. We chose these two angles because: (i) they should both fall in the "eccentricity regime" if our scaling notion is correct; (ii) they are sufficiently separated to produce easily measurable effects; and (iii) we wished to avoid making judgements across the fovea (the 180 deg case), where it could be argued that the high precision of the fovea, or the absence of a fixation target might influence the precision of the judgements. At each eccentricity, the data were collected in blocks of trials counterbalanced across separations (angles). The two-line task is especially suitable for this test, because there is no potentially confounding middle line off the iso-eccentric arc, and because it is not subject to the effects of crowding at
small separations (Yap et al., 1989). The data are part of the data set shown in Figs 9A and B for JT and CN, respectively (new data were collected for DL), and the data for all three observers are plotted as a function of the angle in Fig. 13 (to be discussed below).

Table 3 (top) shows the slope of the data on log-log coordinates. Note that on log-log coordinates the prediction of Weber's law is a slope of 1, whereas our results show mean slopes of 0.023 for observer DL, 0.036 for JT and 0.092 for CN (the mean slope across observers and eccentricities = 0.05 ± 0.15). It is clear that the slopes of the regression lines provide little support for the role of separation in spatial interval judgements at large separations. Not only do our data not support Weber's law (slope = 1), but they are also not consistent with a square root law (slope = 0.5).

We have made similar paired two-line spatial interval discrimination measurements on each of the three observers at two small separations, 7 and 20 deg. The slopes of the straight lines through the data (log-log coordinates) for this condition are shown in the lower portion of Table 3. Note that at small separations the slopes are close to 1 (mean slope across observers and conditions = 0.91 ± 0.06), compatible with Weber's law. These two-line spatial interval data are plotted in Fig. 13 for DL (triangles), CN (squares) and JT (circles), with the thresholds specified as a fraction of the eccentricity, and the separation plotted as the angle between the fixation point and the iso-
Table 3. Two-line interval threshold vs separation slope (log-log)—from Fig. 13

<table>
<thead>
<tr>
<th>Eccentricity</th>
<th>Observer</th>
<th>Large separations (angles = 50 and 120 deg)</th>
<th>Small separations (angles = 7 and 20 deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Mean slope (Across o’s) Weber’s law</td>
<td>Mean slope (Across o’s) Weber’s law</td>
</tr>
<tr>
<td>1.25 deg</td>
<td>DL</td>
<td>-0.28 0.089 -0.004 -0.06 1.0 0</td>
<td>0.94 0.95 0.92 0.94 1.0 0</td>
</tr>
<tr>
<td></td>
<td>CN</td>
<td>0.24 0.179 0.16 0.193 1.0 0</td>
<td>0.91 0.80 0.90 0.87 1.0 0</td>
</tr>
<tr>
<td>5.0 deg</td>
<td>JT</td>
<td>0.11 0.069 -0.048 0.024 1.0 0</td>
<td>0.93 0.80 0.99 0.91 1.0 0</td>
</tr>
<tr>
<td>Mean slope (Across ECC)</td>
<td>0.023 0.092 0.036 0.05 ± 0.15 1.0 0</td>
<td>0.93 0.85 0.94 0.91 ± 0.06 1.0 0</td>
<td></td>
</tr>
</tbody>
</table>

eccentric test lines. There are two important points to note. First, the data of the three observers, and at all three eccentricities are remarkably similar when plotted in these coordinates. Secondly, at the two small angles, thresholds are proportional to separation, and are consistent with the Weber’s law hypothesis shown by the dotted line. On the other hand, at the two larger separations, thresholds at each eccentricity are approx. 0.03 eccentricity, consistent with the spatial grain hypothesis (the solid line in Fig. 13), i.e. they are essentially independent of separation (slope close to 0, Table 3) and are proportional to eccentricity. Finally, it should be noted from Table 1 that for all of the two-line experiments the chi-squared values are better (lower) for the two-mechanism model fit, than for any of the alternative models tested.

Two-line vs three-line thresholds

It is interesting to note that the two-line spatial interval discrimination thresholds are higher than those obtained with three lines at large separations. This is to be expected since the three-line task affords: (i) two opportunities to discern the cue; (ii) a simultaneous reference; and (iii) anticorrelated intervals (when one interval gets bigger the other gets smaller). Thus three-line thresholds should be smaller than two-line thresholds. For the two-line spatial interval task, thresholds were approximately twice as high as the three-line spatial interval discrimination thresholds at large separations. Thus, \( \lambda_{\text{rec}} \) (Table 2) is between about 0.027 and 0.038 for the two-line task, compared to 0.011–0.02 in the three-line task. In order to make a more direct comparison, we have obtained new three-line data at the same (shorter) viewing distance, and with all three lines flashed simultaneously. Figure 14 plots two-line and three-line data obtained under comparable conditions, and to facilitate comparison, the abscissa is the angle subtended by the pair of iso-eccentric test lines and the fixation point. Figure 14A shows the data of JT at 5 deg, and Fig. 14B shows data of DL at 10 deg. Note that for both observers and eccentricities, the three-line thresholds (filled symbols) are only slightly better than two-line thresholds at small separations; however, at large separations (greater than about 40 deg) they differ by about a factor of two. The factor of two improvement for the three-line task is to be expected because the spatial interval stimulus affords twice the information if the observer compares the two intervals. Klein and Levi (1987) presented a derivation showing that the three-line threshold was \( \sqrt{2} \) times smaller than the intrinsic position uncertainty \( \sigma \) and that the two-line spatial interval discrimination threshold was \( \sqrt{2} \) times larger than \( \sigma \). This comparison of two-line vs three-line thresholds is expanded in the Appendix. It is quite surprising that the two-line separation thresholds are as good as they are at large separations, since size constancy and criterion fluctuations might be expected to render the judgements very difficult. We believe that these difficulties are minimized by providing explicit feedback regarding both the direction and the magnitude of the offset following each trial.
The results shown in Fig. 14 suggests that the middle line of the three-line target indeed has an effect upon the thresholds (Levi et al., 1988), but more importantly, they show that the failure of Weber's law at large separations is not a consequence of having a reference feature off the iso-eccentric arc. It is interesting to note that the breakpoint between the Weber regime and the eccentricity regime occurs at a larger separation in the two-line task (about 1/2 to 2/3 of the eccentricity) than in the three-line task (about 1/4 to 1/3 of the eccentricity). This is because
separation is defined differently for the two tasks (see Fig. 2); however, when considered in terms of the angle between the fixation point and the pair of test lines on the iso-eccentric arc, the breakpoint is similar for both the two- and three-line tasks, and occurs at angles of around 20–40 deg (see the section on "The transition from Weber's law to the eccentricity regime").

**Thresholds with a foveal reference line**

In each of the three-line iso-eccentric summary plots (e.g. Figs 3–7), the rightmost datum, representing a separation equal to the radius of the iso-eccentric arc, is obtained with the reference line presented to the fovea. These points are replotted in Fig. 15 for JT (Fig. 15A) and KH (Fig. 15B), and represent the data for separations of 0.625 deg and greater. Data were also obtained at smaller separations, by fixing the observers’ viewing distance at 6.24 m and varying the interline separation on the screen. These data provide an excellent replication of the results of Klein and Levi (1987) and there are several points worth noting: (1) as noted above, the alignment thresholds are lower (better) than the spatial interval discrimination thresholds at each separation/eccentricity; (2) both observers show a sharp discontinuity at a separation between about 0.3 and 0.5 deg in the spatial interval discrimination thresholds. Based upon very similar results, Klein and Levi inferred that spatial interval discrimination thresholds are limited by two mechanisms.

**Position threshold vs eccentricity**

Vision scientists have long been interested in the rate of fall-off of visual function with eccentricity (e.g. Aubert & Foerster, 1857; Weymouth, 1958; Saarinen, Rovamo & Virsu, 1989), and over the past few years, a number of investigators have measured the decline of position acuity with eccentricity (Westheimer, 1982; Levi, Klein & Aitsebaomo, 1985; Beck & Halloran, 1985; Yap et al., 1987a, 1989; Toet, Snippe & Koenderink, 1988; Virsu, Nasanen & Osmovita, 1987; Burbeck, 1988). The decline of visual function with eccentricity can be characterized by the dimensionless factor $E_1$, the eccentricity at which the foveal threshold doubles (Levi et al., 1985; Yap et al., 1987a). To estimate the value of $E_1$ for the eccentricity regime we used several different approaches. First we applied the two-line fit described by Klein and Levi (1987) to the three-line spatial interval data of Fig. 15. Since the data at separations of 0.625 deg and greater in Fig. 15 represent the rightmost point of each iso-eccentric curve, they provide a reasonable estimate of $E_1$ for the eccentricity regime. For JT and KH, the $E_1$ values were $0.47 \pm 0.14$ and $0.44 \pm 0.07$ respectively. For observer DL, $E_1$ was $0.64 \pm 0.08$ (the data are presented in Klein & Levi, 1987). We also refit all the iso-eccentric data from Figs 3–9 at large separations (in the entire eccentricity regime) using the formula: $\text{Th} = k(E + E_1)^{0.45}$. This is the power function which Van Essen et al. (1984) fit to macaque V1 magnification along iso-eccentric contours. This power function provided a better fit to the data than the linear fit [i.e. $\text{Th} = k(E + E_1)$] and the average value of $E_1$ was approx. 0.45. To determine the value of $E_1$ for the eccentricity regime with greater precision will require measuring thresholds on iso-eccentric arcs with smaller radii. For the present data the value of $E_1$ is only important for setting the height of the horizontal line segments shown in each of the data graphs at small eccentricities. The estimated value of $E_1$ is within the range of about 0.4–0.9 deg reported previously for the decline of optimal position acuity with eccentricity (Westheimer, 1982; Levi et al., 1985; Yap et al., 1987a; Klein & Levi, 1987), and is also compatible with the variation in the inverse cortical magnification (V1) with eccentricity found in physiological (Dow et al., 1981 as recalculated by Levi et al., 1985; van Essen et al., 1984) and anatomical (Tootell et al., 1982; also E. Schwartz, personal communication) studies of monkeys and humans (also see Tolhurst & Ling, 1988).

**Relationship to previous (non-iso-eccentric) studies**

Most previous studies of position discrimination in the periphery were non-iso-eccentric. Unfortunately, the results are often inconsistent. Some authors reported that when thresholds for stimuli with large, fixed separations are plotted against eccentricity, there is little or no change in threshold with eccentricity (Beck & Halloran, 1985; Toet et al., 1988; Burbeck, 1988). On the other hand, at small separations, the increase of threshold with eccentricity is precipitous (Westheimer, 1982; Levi et al., 1985; Yap et al., 1987a, 1989). We believe that many of the discrepancies can be understood on the basis of our two-mechanism model. Thus, in separate experiments, we
measured three-line spatial interval discrimination and three-line alignment thresholds for fixed separations at several eccentricities. The fixed separation data were measured at a fixed viewing distance (0.39 m) with lines which were 30' long at all eccentricities, and the position of the stimulus was varied with respect to the fixation point in order to present the targets at different eccentric loci in the lower visual field. Note that this is not iso-eccentric, but is similar to the methods that have been conventionally used to measure position thresholds in peripheral vision (Westheimer, 1982; Levi et al., 1985; Beck & Halloran, 1985; Yap et al., 1987a, 1989; Toet et al., 1988; Virsu et al., 1987; Burbeck, 1988; Palmer & Murakami, 1987). Figure 16 (left) shows the three-line spatial interval discrimination results of observer JT at fixed separations of 10 deg (A) and 1.2 deg (B). In order to facilitate comparison with previous...
Fig. 16. Left: JT’s three-line spatial interval discrimination thresholds at fixed separations of 10 deg (A) and 1.2 deg (B) at various eccentricities in the lower visual field. Note that this experiment was not on an iso-eccentric arc. For the 10 deg separation (A) there was only a very small change in threshold with eccentricity, while at 1.2 deg (B), thresholds increased by about a factor of three between 0 and 5 deg, with little change from 5 to 10 deg. The dotted and dashed lines show the predictions for Weber’s law at 10 and 1.2 deg, respectively. The solid lines show the predictions of our two-mechanism model (see text for details). Right: CN’s three-line alignment thresholds at fixed separations of 10 deg (C) and 2.4 deg (D) at various eccentricities in the lower visual field.

non-isoeccentric experiments, the abscissa shows the eccentricity of the middle line. For the 10 deg separation there was only a slight increase in threshold with eccentricity in agreement with Beck and Halloran (1985), while at 1.2 deg, thresholds increased by about a factor of 2.5 between 0 and 5 deg, and a smaller increase (about a factor of about 1.5) between 5 and 10 deg. The dotted and dashed horizontal lines show the predictions of a two-mechanism model, based upon our iso-eccentric results:

for \( S < 0.5 \cdot \text{ECC} \): \( \text{Th} = \text{SEP} \cdot k_{\text{sep}} \),

for \( S > 0.5 \cdot \text{ECC} \): \( \text{Th} = (\text{ECC} + E_2) \cdot k_{\text{ecc}} \).

Where \( S \) is the separation between the outside lines (the value of \( S = 0.5 \) corresponds to an angle of 30 deg, the point of transition between the Weber’s law and the eccentricity regimes), SEP is the separation between the test and reference lines, \( k_{\text{sep}} = 0.05 \) and \( k_{\text{ecc}} = 0.01 \).
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For the 2.4 deg separation, the eccentricity mechanism limits performance for references at 0, 2.5 and 5 deg, since the separation is small with respect to the eccentricity. However, at 10 deg, the eccentricity mechanism and the Weber’s law mechanism are equally sensitive. For both tasks, it is clear that the two-mechanism predictions (solid line) provides a better fit to the data than Weber’s law.

Our analysis of the fixed separation data suggest that there are two ways in which results like those of Beck and Halloran (i.e. very little variation in thresholds with eccentricity) can be obtained. One is if the separation falls into the Weber’s law regime at each eccentricity tested (i.e. at small separations, \( \theta < \approx 40 \) deg). The other is if the fixed separation falls into the extreme eccentricity regime at each separation tested (i.e. at large separations with \( \theta > \approx 120 \) deg), where, by the Pythagorean theorem, the eccentricity of the test lines does not change much as the eccentricity of the reference is varied.

Alternative explanations and other loose ends

Our data provide strong evidence for a failure of Weber’s law at large separations. In this section we consider: (i) possible artifacts which could contribute to our results; (ii) the generality of our results; (iii) task dependence; and (iv) alternative explanations.

(i) Possible artifacts—One potentially serious methodological problem in our three-line experiments is the presence of a reference line which is not on the iso-eccentric arc. However, the two-line experiments give qualitatively similar results, and in those experiments both lines were presented on the iso-eccentric arc. Thus, the two-line experiments show that the failure of Weber’s law at large separations was not a consequence of the middle line.

The edges of the screen might also provide a reference whose effect can change for different separations. We believe that the screen edges are unlikely to provide a useful reference in our experiments. Firstly, we jittered the position of the stimuli relative to both the edge and the fixation point by an amount several times the threshold. Secondly, in the three-line experiments, halving the viewing distance had no significant effect on the thresholds (see Fig. 1). Thirdly, for all the two-line experiments, and the half distance control in the three-line experiments, the field size was approx. three times the largest separation, so the edges were quite
(ii) **Generality of the results**—Our data suggest that Weber's law fails when the separation of the outside lines is larger than about half of the eccentricity for two-line and three-line spatial interval and alignment judgements, and over a range of eccentricities between 0.625 and 10 deg. Thus, it appears that the failure of Weber's law is rather general. Moreover, using quite different methodology, Yap and Burbeck (1988) have observed a similar flattening at large separations in two-line iso-eccentric spatial interval discrimination experiments. There are, however, exceptions. Recently, Palmer and Murakami (1988) measured two-line spatial interval discrimination, and failed to observe a flattening. We have no explanation for their results; however, it is worth noting that their thresholds at large separations are about 4–5 times higher (worse) than ours. Morgan and Watt (1989) have also failed to observe a flattening at large separations using an iso-eccentric arc length discrimination task. Here too, their thresholds at large separations were 4–5 times higher (worse) than ours, and elsewhere (Levi & Klein, 1989) we argue that their task involves a complicated computation of arc length. Rather than tapping the sensory limits imposed by the target eccentricity, their thresholds may be constrained by the cognitive demand of the task, or by the difficulty of reconstruction. Thus, it is clear that there may be tasks which do not follow our two-mechanism hypothesis. Our point is that for precise spatial discrimination, both the target separation and its eccentricity may impose sensory limitations on performance.

(iii) **Task dependence**—While each of our tasks is qualitatively similar in showing a range of large separations (angles greater than 30–40 deg) where thresholds are more or less independent of separation, and a range of smaller separations where thresholds are proportional to separation, there are quantitative differences among the tasks. For example, three-line thresholds are lower (better) than two-line thresholds at large separations. This point is examined in detail in the Appendix. Similarly, alignment thresholds are lower than spatial interval thresholds. For example, at small separations, where threshold = SEP * \( k \) the value of \( k \) is \( \approx 0.05 \) for spatial interval discrimination and \( \approx 0.03 \) for alignment. The superiority of alignment thresholds is also evident at large separations, where thresholds = ECC * \( k \). The value of \( k \) (the fraction of eccentricity) was between 0.01 and 0.02 for three-line spatial interval discrimination, while for three-line alignment it was as low as 0.007. Some of these differences, and the role of memory vs explicit references, are explored in detail in the Appendix.

(iv) **Alternative explanations**—Weber's law is pervasive in sensory physiology, so the failure of Weber's law at large separations is surprising. Neither a square root law, nor a single fixed exponent provide as good a fit to the data as the two-mechanism model. However, it seems important to consider alternative explanations. One possibility is based on the known anisotropies in position discrimination (Yap, Levi & Klein, 1987b) which could influence performance. Could these anisotropies account for the flattening observed at large separations? Consider the spatial interval task. For large separations, the direction of offset is radial to the line joining the target to the fixation point, while for small separations, the offset is tangential or iso-eccentric. Yap et al. (1987b) showed that bisection thresholds were better for iso-eccentric (tangential offsets) than for radial offsets. Thus, these anisotropies would produce the opposite result; i.e. the directional anisotropy would make performance worse, rather than better at large separations. Moreover, the situation is reversed for the alignment task, i.e. for large separations the offset is tangential while at small separations it is radial, yet the same result is obtained. Thus, it seems unlikely that our results can be simply explained on the basis of anisotropies. Levi et al. (1988) also showed that the failure of Weber's law at large separations could not result from the meridional variations in acuity. Moreover, the experiments with the rotated display suggest that the failure of Weber's law at large separations is not a consequence of making comparisons across the vertical midline. An alternative explanation is that the horizontal and vertical meridians are favored over other visual field locations; since the largest separations at each eccentricity always fall along one of these meridians (depending on the orientation of the stimuli), there may be a confounding of separation and meridian. We do not think it likely that our results can be explained on the basis of performance on versus off the horizontal and vertical meridians. Our results suggest that thresholds are more or less constant when the angle between the fixation point and the iso-eccentric
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The present results are consistent with the suggestion that distance judgements are limited by at least two factors: (1) a Weber computation where threshold = SEP \* k. In this regime position thresholds are proportional to the separation of the features, and depend only slightly on eccentricity. This is essentially Weber's law for position. A number of models have been proposed to account for the general principle of Weber's law (see for example Laming, 1986), and for Weber's law for position (e.g. Klein & Levi, 1985; Wilson, 1986); (2) an eccentricity regime with a precision of about ECC \* k. In this “eccentricity” regime, position thresholds are independent of separation, and depend only upon the target eccentricity. In this regime, threshold for alignment of widely-separated line targets was about 0.008 times the target eccentricity. These thresholds are quite acute, and thus likely represent sensory (rather than cognitive) limitations to encoding spatial position. It is interesting to note that at each eccentricity the alignment thresholds were lower (better) than the spatial interval discrimination thresholds (compare Figs 3 and 4 with Figs 6 and 7; also note the model fit values in Table 2).

To the degree that alignment and interval thresholds are different at large separations, the simple isotropic cortical grain hypothesis cannot be supported (Klein & Levi, 1987). Our results imply that the “cortical ruler” used for interval judgements and the “cortical ruler” used for alignment have different sensitivities. The important point, however, is that our iso-eccentric data show that for widely-separated lines, both interval and alignment thresholds are proportional to the eccentricity of the lines, and depend very little on their separation. When the separation is large with respect to the eccentricity, so that the cortical representations of the target features are separated by several millimeters of cortical distance, the judgement of distance is similar to a distance measurement using a ruler on the cortex, in that the error of measurement is independent of the separation.
between objects. This large separation regime, is reminiscent of Lotze's notion of local signs (localzeichnen; Lotze, 1884)—i.e. that the direction in which an object is perceived is an intrinsic property of the visual system. It is interesting to speculate about the failure of Weber's law for position at wide separations considering it's generality in so many other sensory judgements. One possibility is that spatial position is unique in it's precise retinotopic mapping, so that the observer has access to the local sign of each point in the visual field, independent of the separation of the points. In this regime, positions thresholds are essentially independent of target luminance (Bedell, Johnson & Barbeito, 1985; Yap et al., 1989), contrast (Morgan & Regan, 1987; Toet & Koenderink, 1988), timing (Yap et al., 1987a), spatial frequency (Burbeck, 1987; Toet & Koenderink, 1988) and are tolerant to quite severe blurring (Toet et al., 1988), as might be expected if the threshold depends upon the sampling properties of the visual system. It is of particular interest to note that for large separations (greater than about 30 min) two-line Vernier acuity is as good when the lines are presented dichoptically (i.e. one to each eye), as when they are presented monocularly (McKee & Levi, 1987), suggesting that position judgements for widely separated targets are limited at the cortex. Our results suggest that this "position sense" is quite acute, as it should be for accurately guiding fixational eye-movements to peripheral targets, and for acute discrimination of variations in size or angle.

Acknowledgements—We thank Jenny, Cindy and Kelly for their many hours of observations, and Tom Banton for helpful suggestions on an earlier version of this manuscript. This work was supported by grants RO1 EY01728 and RO1 EY04776 from the National Eye Institute.

REFERENCES


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APPENDIX

Relationship of Position Thresholds to Underlying Sampling Noise

This appendix is an expansion of the discussion by Klein and Levi (1987) relating position thresholds to the standard deviation of the underlying sampling uncertainty. Klein and Levi (1987) discussed the three-line task in which the central line had no uncertainty. In this appendix both the two-line and three-line tasks are considered and are generalized to include reference uncertainty. To introduce our formalism consider the two-line size paradigm. Suppose the two lines are at positions $x_1$ and $x_2$ and the memorized reference separation is $M$. The observer must judge whether $\delta = x_1 - x_2 - M$ is less than, equal to, or greater than zero. The accuracy of the judgement depends upon the variance with which $x_1$, $x_2$, and $M$ are encoded and processed. Every measurement is associated with a standard error $\sigma$ and it is the goal of this appendix to show how the standard error of each individual position measurement (the intrinsic spatial uncertainty) affects position thresholds determined psychophysically.

The variance of $\delta = x_1 - x_2 - M$ is:

$$\text{var}(\delta) = \sigma_1^2 + \sigma_2^2 + \sigma_M^2 - 2(\sigma_1 \sigma_2 - \sigma_1 \sigma_M - \sigma_2 \sigma_M) = \text{var}(\delta) - \text{var}(\delta - M).$$

where $\sigma_1^2$, $\sigma_2^2$ and $\sigma_M^2$ are the variances of $x_1$, $x_2$ and $M$; $\sigma_1 \sigma_2$ is the covariance between $x_1$ and $x_2$ and $\sigma_1 \sigma_M$ and $\sigma_2 \sigma_M$ are the covariances between the memorized reference width and the observed separation $x_1 - x_2$. The covariance terms can be significant. A fluctuation in the judged “straight-ahead” direction would cause a correlated shift in both $x_1$ and $x_2$, leading to a positive value of $\sigma_1 \sigma_2$. Similarly, fluctuations in a “size constancy” mechanism can produce correlations between $M$ and $x_1 - x_2$, since an increase in perceived depth might produce an increase in both $M$ and $x_1 - x_2$.

This appendix explores several approaches for quantifying $\text{var}(\delta)$. $\text{var}(\delta)$ is directly related to the psychophysical threshold, $Th$, by $Th = 0.675 \times \text{var}(\delta)^{0.5}$. The factor of 0.675 arises because for the present experiments, threshold was defined at the 75% point, corresponding to a $d'$ of 0.675 rather than a $d'$ of unity.

To first order, $\text{var}(\delta)$ is a constant plus a term that is proportional to the separation $s$:

$$\text{var}(\delta) \propto a_1 + b_1 s + a_M + b_M s,$$

where

$$a_1 + b_1 s \approx \sigma_1^2 + \sigma_2^2 - 2(\sigma_1 \sigma_2),$$

and

$$a_M + b_M s \approx \sigma_M^2 - 2(\sigma_1 \sigma_M).$$
a Weber's law mechanism (the separation regime) and a local sign mechanism (the eccentricity regime). For the Weber's law mechanism the terms in \( \text{var}(\delta) \) that are proportional to \( s \) are larger than the constant terms since in the Weber's law regime thresholds are well fit by a straight-line that is proportional to separation with a zero intercept (Klein & Levi, 1987); so equation (A2) can be approximated by neglecting the constant terms:

\[
\text{var}_e(\delta) \approx (b_{\text{ne}} + b_{\text{we}}) s.
\]  

(A5)

Similarly, for the local sign (eccentricity-dependent) mechanism, the data of the present paper indicate that the constant terms are larger than the terms proportional to \( s \) (if anything, thresholds decrease with \( s \)), so the terms proportional to \( s \) can be neglected:

\[
\text{var}_w(\delta) \approx (a_{\text{ne}} + a_{\text{we}}),
\]  

(A6)

where the subscript \( e \) stands for the eccentricity regime, and the subscript \( w \) stands for the Weber's law (separation-dependent) regime.

This appendix does not examine the Weber's law regime, which is thought to involve size-tuned filters. That regime, which requires assumptions about the sensitivities and bandwidths of the underlying mechanisms, is treated in Klein and Levi (1985).

This appendix examines the eccentricity regime which becomes easy to analyze if we assume that equation (A6) can be rewritten in the form:

\[
\text{var}(\delta) = \sigma_i^2 + \sigma_s^2 + \sigma_w^2.
\]

(A7)

This appendix examines the eccentricity regime which becomes easy to analyze if we assume that equation (A6) can be rewritten in the form:

\[
\text{var}(\delta) = \sigma_i^2 + \sigma_s^2 + \sigma_w^2.
\]

(A7)

The variance of the comparison of two intervals is given by the formula:

\[
\text{var}(\delta) = \sigma_i^2 + \sigma_s^2 + \sigma_w^2.
\]

Equation (A7) differs from equation (A1) because the covariance terms are absent and because the standard errors now have primes. The term \( \sigma'_s \) does not necessarily equal \( \sigma_s \) since \( \sigma'_i \) can also contain a part of the covariance term. This allows \( \sigma'_s \) to be task dependent. In particular, consider the data in Fig 10 for two-line alignment and two-line spatial interval discrimination, where the separation is 7 deg. This point is on the 45 deg meridian so there is a symmetry between vertical offsets (for the alignment task) and horizontal offsets (for the spatial interval task). Symmetry arguments would predict that both tasks should have approximately equal thresholds. Figure 10, however, shows that the position threshold is about 40% better for the alignment task than for the spatial interval task (also compare Figs 3 and 4 to Figs 6 and 7 for similar results on two other observers on pairs of three-line tasks). If the covariance term had been omitted then thresholds for the orientation and size tasks should be equal since only the local spatial grain matters. Our use of \( \sigma' \) rather than \( \sigma \) provides the flexibility to have the value of \( \sigma' \) be task dependent. Another way of saying it is that the ruler that is used for size judgements is noisier than the ruler used for alignment judgements.

Our data also shows that the magnitude of the spatial standard errors \( \sigma'_s \) and \( \sigma'_s \) depends upon the location of \( x_1 \) and \( x_2 \) in the visual field. These errors are approximately proportional to the eccentricity of the stimulus. This eccentricity dependence of \( \sigma' \) is discussed in the text.

Having developed the formalism, it is now possible to express the variance of \( \delta \) for both the two-line and the three-line paradigms. For the two-line case the lines are at positions \( x_1 \) and \( x_2 \) and we assume that a random position increment is given to \( x_1 \). For the three-line case the test lines are still at positions \( x_1 \) and \( x_2 \), but now a reference comparison line is introduced at the midpoint \( x_c \), and a random position increment \( \delta \) is given to the two outside lines. The increment to the outside lines is the same as an increment of \(-\delta \) given to the middle line. Notice that adding the increment to the middle line increases one of the intervals and decreases the other interval, so if the observer judged the relative size of the two intervals, the threshold for the three-line task could be a factor of two smaller than for the two-line task (such as in the same configuration but with one of the outside lines missing). However, the relationship between the two-line and three-line tasks is more complicated since it also depends upon the standard error of the memory standardized \( M \) and the standard error of the comparison line \( x_c \). According to the above formalism the variance of \( \delta \) can be written taking all these factors into account:

\[
\text{var}(\delta) = \sigma_i^2 + \sigma_s^2 + \sigma_w^2.
\]

(A7)

Equation (A8a) is the same as equation (A7). Equations (A8b) and (A8c) give thresholds for the three-line task and are based on comparing either one or both of the two intervals to the memory standard. The square root of 2 in equation (A8c) comes from averaging two independent signals. Equation (A8d) would result if thresholds were based on a comparison between the two intervals in the bisection task.

Let us assume \( \sigma'_1 = \sigma'_2 = \sigma' \), because of symmetry. To further simplify the comparison of two-line and three-line task we define the ratio \( r \) as the standard error divided by the standard error of the outside test lines:

\[
r_M = \frac{\sigma'_M}{\sigma'}, \quad r_w = \frac{\sigma'_w}{\sigma'}.
\]

(A9a)

(A9b)

For the three strategies in the three-line tasks (equations 88b-d), the relative thresholds for the two-line to three-line cases are, respectively:

(A10a)

(A10b)

(A10c)
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Table A1. Ratio of two-line to three-line thresholds

<table>
<thead>
<tr>
<th></th>
<th>( \frac{r_{m}}{r_{c}} &lt; 1 )</th>
<th>( \frac{r_{m}}{r_{c}} = 1 )</th>
<th>( \frac{r_{m}}{r_{c}} &gt; 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A)</td>
<td>Equation (A10a) ( \sqrt{2} )</td>
<td>1 ( \sqrt{1.5} ) 1</td>
<td>1 ( \sqrt{1.5} ) 1</td>
</tr>
<tr>
<td>(B)</td>
<td>Equation (A10b) ( 1 ) ( \sqrt{2} ) ( 0.5 \sqrt{3} ) ( \sqrt{2} ) ( \sqrt{2} )</td>
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<tr>
<td>(C)</td>
<td>Equation (A10c) ( 2 ) ( \sqrt{3} ) ( 2 ) ( \sqrt{2} ) ( \sqrt{2} ) ( \sqrt{2} )</td>
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</table>

Depending on the magnitude of the memory uncertainty \( r_{m} \) and the middle-line reference uncertainty \( r_{c} \), different values for the ratio of two-line to three-line thresholds are obtained as shown in Table A1. If the memorized reference \( M \) has a large uncertainty \( \sigma'_{m} \), then the two-line threshold can be arbitrarily worse than the three-line threshold. For example, in a dark room if there are minimal cues to the distance of the object, it is possible for the perceived size of the two-line target to fluctuate widely and for the two-line threshold to be many times the three-line threshold.

For the two-line vs three-line spatial interval discriminations at large separations (in the eccentricity regime), our data show that the two-line thresholds are about two times worse than the three-line thresholds. The only possible mechanism that can lead to this large ratio is option C of the Table A1 where the two intervals of the three-line task are compared to each other (consistent with the subjective reports of the subjects). The constraints on \( \sigma'_{m} \) and \( \sigma'_{c} \) are weak since most combinations of these parameters would give acceptable values for the ratio of two-line to three-line thresholds. The values in the table do indicate, however, that \( \sigma'_{m} \) should be larger than \( \sigma'_{c} \).

For the two-line vs three-line orientation task, our data show that the ratio is about \( \sqrt{2} \). The values in equation (A10) indicate that there are two ways of achieving the \( \sqrt{2} \) ratio. Either option A where a single interval is compared to a standard with minimal uncertainty (column 1) or the two intervals are compared (option C), but the memory and comparison stimulus have the same uncertainty as the outside test lines (column 4).