SPATIAL INTERVAL DISCRIMINATION WITH BLURRED LINES: BLACK AND WHITE ARE SEPARATE BUT NOT EQUAL AT MULTIPLE SPATIAL SCALES

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Abstract—We used Gaussian blurred lines of same- and opposite-polarity to measure the effects of blur on 3-line spatial interval discrimination (bisection). The results of our experiments can be summarized as follows. Spatial interval discrimination (3-line bisection) thresholds are proportional to the separation of the lines (i.e. Weber’s law). At the optimal separation, spatial interval discrimination thresholds for same-polarity lines represent a “hyperacuity” as small as 2 sec arc. For same-polarity Gaussian blurred lines, over a wide range of the blur standard deviations (σ), the optimal threshold occurs when the separation is approx. 2σ, and the optimal threshold is about 0.02σ, or a Weber fraction (Δ/s) of 0.01. For opposite-polarity lines, under conditions where same-polarity stimuli yield the best thresholds (at a separation ≈2σ), spatial interval thresholds are an order of magnitude worse than that for same-polarity lines, suggesting that the localization of stimuli of opposite-polarity is much worse than that of same-polarity stimuli over a wide range of spatial scales. At large separations, greater than about 5σ, spatial interval discrimination thresholds are more or less independent of both contrast and polarity. While hyperacuity is generally thought of in terms of the tiny spatial thresholds which are obtained at small separations with stimuli comprised of thin lines, the present results, and those of others, suggest that for same-polarity stimuli, hyperacuity thresholds are a general property of the visual system, occurring at many spatial scales. The present results also suggest that the poor localization of opposite-polarity lines occurs at multiple spatial scales, when the line separation is less than about five times the stimulus spread. We consider several models which can account for particular features of our data.

INTRODUCTION

With foveal fixation human observers can judge relative position with extraordinary precision. Under optimal conditions, position thresholds may be an order of magnitude smaller than the size or spacing of foveal cones. Westheimer (1975), coined the term “hyperacuity” to describe a class of spatial judgements in which thresholds for discriminating the relative positions of targets are typically 3–5 sec arc, much smaller (better) than those obtained for resolving them.

A striking aspect of position acuity is that the thresholds are remarkably robust to changes in the stimulus configuration. Hyperacuity thresholds of 3–6 sec arc are obtained for a variety of tasks including Vernier acuity with lines and dots (Westheimer & McKee, 1977a), orientation discrimination (Sullivan, Oatley & Sutherland, 1972; Westheimer & McKee, 1977a), spatial interval discrimination (i.e. separation discrimination and bisection (Volkmann, 1863; Westheimer & McKee, 1977a; Hirsch & Hylton, 1982; Watt & Morgan, 1983)) and phase discrimination (Badcock, 1984). Moreover, Westheimer and McKee (1977a) showed that observers could reliably discriminate between a pair of features separated by 3.0 min arc and a similar pair separated by 3.1 min arc regardless of whether the spatial interval was delimited by a pair of bright lines, a pair of dark lines, or a bar. The robust nature of position discrimination thresholds to stimulus configuration suggests that there may be a fundamental limitation on precise position judgements (Westheimer & McKee, 1977a).

In order to understand the nature of the mechanisms leading to precise spatial discriminations, it is informative to ask what stimulus conditions degrade position judgements. Recently Levi and Westheimer (1987) showed that 2-line spatial interval discrimination is markedly degraded when the lines delimiting the interval have opposite-polarity. Under precisely the
conditions where performance with same-polarity stimuli is best, i.e. at small separations, performance with opposite-polarity stimuli is quite poor. At "large" separations (>4-6 min) the precision of spatial interval discrimination is independent of polarity. Similar observations of degraded performance with opposite-polarity stimuli have also been reported for Vernier acuity (Murphy, Jones & Van Sluyters, 1988) and by Morgan (personal communication, 1986). Based upon their polarity data as well as previous studies of spatial interference (Westheimer & Hauske, 1975; Levi, Klein & Aitsebaomo, 1985; Yap, Levi & Klein, 1987, 1989), spatial integration (Westheimer & McKee, 1977b), and the zone of "attraction" (Badcock & Westheimer, 1985), Levi and Westheimer (1987) postulated that stimuli that fall within a spatial processing module of about 4 min arc are subject to different processing, and thus different constraints than stimuli whose features fall in separate modules.

Recently, we (Levi & Klein, 1990a) used Gaussian blurred lines to study the effects of Gaussian blur on 2-line resolution and 2-line spatial interval discrimination. Our results, in agreement with those of Toet, Van Eekhout, Simons and Koenderink (1987) suggest a striking "scale invariance" in spatial discrimination at different levels of blur. We found that over a wide range of stimulus Gaussian standard deviations (σ): (i) the resolution threshold was approximately equal to σ; (ii) the optimal position acuity threshold occurred at a separation of about 2σ; and (iii) at separations larger than 2-3σ, thresholds followed the familiar Weber's law relationship with separation (Fechner, 1860, Volkmann, 1863; Klein & Levi, 1985, 1987; Levi, Klein & Yap, 1988; Levi & Klein, 1990b). Each of these features of spatial interval processing are readily explained on the basis of a simple local contrast model (Klein & Levi, 1989).

The scale invariant nature of 2-line separation discrimination leads to the question of whether the polarity-dependent nature of fine spatial interval judgements reflects the limitation of a processing module of about 4 min arc, or whether it is a more general feature of pattern processing which can occur at different spatial scales. Toet (1987) measured differential spatial displacement (similar to 3-dot Vernier acuity) with Gaussian blobs of same- and opposite-polarity, and found no difference in thresholds for the two conditions. However, Toet's stimuli were very low contrast, and very widely separated (i.e. the separation was equal to 14 × σ). In the present study we measured spatial interval discrimination (3-line bisection) using Gaussian blurred lines of same- and opposite-polarity for a range of separations and standard deviations.

METHODS AND PROCEDURES

Stimuli

The stimuli were three horizontal lines each of whose luminance profile was Gaussian blurred according to $G(x) = \exp(-x^2/2\sigma^2)$, where σ is the standard deviation of the Gaussian. Figure 1 shows examples of our stimuli. Each row shows a different separation between the lines, equal to (from top to bottom) 1, 2, 3 and 10σ. The left and center columns show same-polarity stimuli, dark and light respectively. The right column shows opposite-polarity stimuli, i.e. a light "test" line in the middle, and dark outer reference lines. (Switching the polarities of test and reference lines gave identical results.) Each stimulus in Fig. 1 has the same offset of the middle line (0.2σ). Figure 1 shows clearly one of the main features of our data. For separations of 2 and 3σ, the offset is quite obvious in the same-polarity stimuli, but not in the opposite-polarity stimuli. Another aspect of our results which can be seen in Fig. 1 is that at a separation equal to σ, the situation is reversed, so that the offset is more easily detected in the opposite-polarity stimulus. These observations hold over a wide range of viewing distances. Note that the separation, S, is specified as half of the separation between outer "reference" lines, i.e. it is the separation between the bisection point and the peak of one of the outer reference lines. The stimuli were generated by a programmable arbitrary function generator under computer control, and were presented on a Joyce CRT with a white, P4 phosphor, and mean luminance of 115 cd/m2. The function generator was synchronized to the fast sweep of the Joyce, and two programmable attenuators in series provided excellent control of stimulus contrast and timing. Each Gaussian was truncated at 4 standard deviations (SD). In order to minimize temporal transients, the stimuli were ramped on over 300 msec, remained at a plateau for 600 msec, and were ramped off over 300 msec. We used a long stimulus duration, because spatial interval thresholds are best with long viewing durations (Yap et al., 1987).
Figure 1 is printed overleaf.
Fig. 1. Examples of our Gaussian blurred stimuli. Each row shows a different separation between the lines, equal to (from top to bottom) 1, 2, 3, and 10°. Note that the separation, S, is specified as half of the separation between the outer “reference” lines, i.e., it is the separation between the bisection point and the peak of one of the outer reference lines. The left and center columns show same-polarity stimuli, dark and light respectively. The right column shows opposite-polarity stimuli, i.e., a light middle “test” line, and dark outer reference lines. Each stimulus is shown with the same offset of the middle line, equal to 0.2°.
Spatial interval discrimination

Unless otherwise specified, the peak contrast of each line was approx. 33%; however, in order to minimize contrast cues to resolution or separation, we introduced a contrast jitter of 30% from trial to trial. Absolute position cues were eliminated by jittering the position of the entire stimulus. In order to achieve a wide range of line separations and standard deviations, we fixed the Gaussian standard deviation at 10 pixels, and varied the separation of the lines on the screen from 10 to 100 pixels (note that we specify separation from the bisection point to the center of one reference line). The standard deviation was varied by changing the viewing distance from 0.5 m ($\sigma = 15.5$ min) to 7.7 m ($\sigma = 1$ min). This strategy varied the line length and the screen size in proportion to $\sigma$. At 7.7 m, the lines were approx. 1 deg long, and the vertical extent of the screen was 1.5 deg. For stimuli with larger standard deviations, all dimensions were scaled proportionally.

Psychophysical method

Thresholds for 3-line spatial interval discrimination (bisection) were measured using a self-paced rating-scale method of constant stimuli. An experimental run consisted of 125 trials (preceded by 10–20 practice trials). On each trial the middle test line was presented in one of five randomly chosen vertical positions, equally spaced about, and including the bisection point. The observer's task was to judge the position of the test line relative to the bisection point by giving numbers from -2 to +2. This self-paced method of constant stimuli with multiple responses has been described elsewhere (Levi & Klein, 1983). Between runs we varied the separation, polarity and standard deviation of the stimuli. Thresholds for spatial interval discrimination were obtained by calculating a maximum-likelihood estimate of the $d'$ values for each stimulus and interpolating to a $d' = 1$, equivalent to the 84% correct level. To compare the present thresholds to those specified at 75% correct, our thresholds should be multiplied by 0.675. Thus the present thresholds, with much lower contrast lines, no flanks, and specified at $d' = 1$ are quite good.

Results

Figure 2 plots spatial interval discrimination thresholds as a function of separation for lines with $\sigma = 1$ min arc, for each of the three observers. In this, and all subsequent figures, unless otherwise specified, solid circles indicate that the stimuli were same-polarity dark lines; open circles indicate same-polarity bright lines; squares indicate opposite-polarity lines. There are several points to note in Fig. 2. Consider the dark line same-polarity data (0). The same-polarity data follow the familiar V-shaped function of separation previously noted for 3-line bisection with thin lines (e.g. Klein & Levi, 1985). As the separation increases from 1 to between 2 and 3 min, thresholds decline dramatically, so that at the optimal separation, threshold for BJ and DL is just over 2 sec arc, and for CN, it is about 3.5 sec arc. Klein and Levi (1985) went to extreme lengths to achieve bisection thresholds of under 1 sec arc. For example, they used extremely high contrast, bright lines, flanked by optimally positioned flanking lines. They also specified their thresholds at $d' = 0.675$. Thus the present thresholds, with much lower contrast lines, no flanks, and specified at $d' = 1$ are quite good.

Observers and conditions

Three highly practiced observers participated in the experiments. All observers had normal binocular vision and corrected-to-normal visual acuity in each eye. DL and BJ are authors. CN, an optometry student well practiced in psychophysics, was naive as to the purpose of the experiments. Viewing was binocular and with natural pupils.

Calibration

Stimulus luminance and contrast ($\Delta L / L_{\text{max}}$), were calibrated at regular intervals throughout the duration of these experiments, using a Pritchard Spectra Photometer.
Fig. 2. Spatial interval discrimination thresholds vs separation for lines with $\sigma = 1 \text{ min arc}$, for each of the three observers. Solid circles indicate that the stimuli were same-polarity dark lines; open circles indicate same-polarity bright lines; squares indicate opposite polarity lines. The 45 deg line illustrates a Weber fraction of 0.02, and it falls quite close to the data. Note the shape of the opposite-polarity function is quite different from that for same-polarity lines. Over the range of separations tested, it is almost an inverted version of the same-polarity function, so that at separations between 2 and about 4 min, thresholds are markedly elevated.

The squares in Fig. 2 show thresholds for opposite-polarity lines. The shape of the opposite-polarity function is quite different from that for same-polarity lines. Over the range of separations tested, it is almost an inverted version of the same-polarity function, and there are three main points to note: (i) at the largest separation, 10 min, thresholds are similar to those for same-polarity lines; (ii) at separations between 2 and about 4 min, thresholds are markedly elevated. This is similar to the 2-line spatial interval results of Levi and Westheimer (1987), although the difference between the same- and opposite-polarity thresholds is somewhat larger for our bisection task; (iii) at the smallest separation, 1 min, thresholds for opposite-polarity lines are actually better than those for same-polarity lines. The reason for this marked improvement in thresholds can be deduced by inspecting Fig. 1.

When the lines are completely overlapped (separation equals $\sigma$, shown at the top of Fig. 1), an upward shift of the middle bright line produces a noticeable difference in the contrast of the upper and lower reference lines. For this condition (separation equals $\sigma$), our observers all reported using this contrast cue. Note that Levi and Westheimer (1987) never tested at separations less than 2.2 min. To summarize, the results of Fig. 2, for same-polarity lines, thresholds are best when the separation, $S$, equals $2\sigma$. At larger separations, thresholds follow Weber’s law. Thresholds for same- and opposite-polarity lines are similar when $S$ is greater than about 5 $\sigma$. For $S$ between about 2 and 4 $\sigma$, thresholds for opposite-polarity lines are markedly elevated. For $S = \sigma$, thresholds for same-polarity lines are markedly degraded, and thresholds for opposite-polarity lines are lowest. As we will show below, this basic pattern of results is repeated at each value of $\sigma$; however, the data at $\sigma = 1 \text{ min}$ will be seen to also differ quantitatively from those obtained at larger stimulus blurs, probably due in part to the intrinsic blur of the visual system, and to the lower effective contrast of the stimuli.
**The effect of stimulus blur**

Figure 3 shows data for each observer for a small and a large standard deviation. In this, and subsequent figures, the size of the symbol is proportional to the stimulus standard deviation. As before, open and solid circles are for light and dark same-polarity lines, and squares are for opposite-polarity lines. The dot-dashed lines show a constant Weber fraction of 0.02. The same-polarity data follow a V-shaped function of separation with both the optimum threshold and the optimum separation increasing as σ increases. For each σ tested, opposite-polarity stimuli produce markedly elevated thresholds for separations from 2 to about 4 times σ. For separations 5–10 σ, same- and opposite-polarity stimuli of the same σ have similar thresholds. At each value of σ tested, opposite-polarity stimuli gave lower thresholds than same-polarity stimuli when σ equals the separation.

This can be seen more clearly in Fig. 4, where the data of Fig. 3, as well as data at several other standard deviations have been plotted. Now, both the abscissa and the ordinate are expressed in units of the stimulus standard deviation, i.e. they were divided by σ. Figure 4 shows that in many respects the results are scale-invariant, i.e. when threshold and separation are expressed in units of the blur standard deviation, the results are qualitatively similar to the σ = 1 min results of Fig. 2.

As noted above, the data for σ = 1 min differ quantitatively in several respects from the data at larger standard deviations. Firstly, differences between thresholds for dark and bright same-polarity stimuli are diminished or eliminated at larger values of σ. Secondly, for separations greater than about 2 σ the σ = 1 min data are compatible with a Weber fraction of about 0.02 (shown by the dot-dashed line); however, at larger standard deviations, there appears to be some facilitation between separations of 2 and 3 σ, so that thresholds at the optimum approach Weber fractions of 0.01 (this can be seen more easily in Fig. 5). Thus, in σ units, the optimal threshold for stimuli with σ > 2 min are approx. 0.02 σ (since the optimal separation is 2 σ, this leads to a Weber fraction of 0.01). However, the data with σ = 1 min (smallest circles) have an optimum threshold of about 0.04 σ. We believe that the slightly degraded performance with 1 min lines is due to the visual system's intrinsic blur, which is similar in magnitude to the blur of the lines (Levi & Klein, 1990a), and to the lower effective contrast of the 1 min lines.

![Fig. 3. Summarizes data for each observer at two very different standard deviations. Open and solid circles are for light and dark same-polarity lines, and squares for the opposite polarity lines; σ is designated by symbol size. The smallest symbols show σ = 1 min (DL and BJ) or σ = 2 min (CN); the largest symbols show σ = 7.5 (BJ), 15 (CN) or 15.5 min (DL). Each set of curves is similar to those of Fig. 2; however, as σ increases, both the optimum threshold and the optimum separation increase.](image-url)
Fig. 4. The data of Fig. 3, and data at other standard deviations have been replotted with both the abscissa and the ordinate expressed in units of the stimulus standard deviation, i.e. they were divided by $\sigma$. This figure shows that in many respects the results are similar at each level of stimulus blur; i.e. the results are scale-invariant (CN's 1 min data have been omitted for clarity).

Figure 5 replots the data of Fig. 4 in yet another form. In Fig. 5, the ordinate is the threshold divided by the separation (i.e. the Weber fraction, obtained by dividing the ordinate of Fig. 3 or 4 by its abscissa). The abscissa is the stimulus standard deviation divided by the separation, i.e. it is obtained by taking the reciprocal of $\sigma$. An abscissa value of 0.1 indicates that the $S = 10 \sigma$, i.e. large separations are represented on the left-hand side in Fig. 5. The advantage of reploting the data in this format, is that it demonstrates graphically the effect of varying $\sigma$. For same-polarity stimuli at all scales, thresholds are about 0.02 for $\sigma < 0.3 S$ (the dot-dashed lines show a Weber fraction of 0.02). At $\sigma = 0.5 S$, the Weber fraction improves by about a factor of 2. This facilitation is clear for all stimulus blurs except $\sigma = 1$ min. For $\sigma > 0.5 S$, thresholds are sharply degraded by stimulus blur.

The effect of contrast

In order to assess the effect of contrast, we measured spatial interval discrimination thresholds for several separations and $\sigma$'s as a function of contrast. Figures 6–8 show the effects of contrast for observers DL and CN. In each of these figures the abscissa specifies the mean contrast; however, from trial to trial, the contrast was varied randomly by 30% of the nominal value. Figure 6A shows data for DL for $\sigma = 1$ min. The small and large solid circles are for dark lines with separations equal to 2 and 10 min respectively. Note that for dark lines with separation equal to 2 min, contrast has a strong effect upon thresholds. For contrasts below about 10%, thresholds decrease in proportion to contrast (the solid line shows a slope of $-1$). As contrast increases above about 10%, thresholds continue to improve, but at a somewhat slower rate. For a separation of 10 min the contrast dependence is much less, so that a 20-fold increase in contrast only improves bisection thresholds by about a factor of two. The open squares show the effects of contrast on opposite-polarity stimuli with a separation of 2 min. It is interesting to note that the opposite-polarity stimuli have a quite different contrast dependence than same-polarity stimuli of the same 2 min separation. Interestingly, the thresholds and the shape of the curve are similar to that of same-polarity stimuli with a large separation. This figure also shows that the stimulus contrast has an influence on the difference between same- and opposite-polarity thresholds. At the lowest contrast tested, opposite-polarity thresholds were about a factor of two higher than same-polarity thresholds for a separation of 2 min. However, at the highest contrast level, the difference is about a factor of 6. Figure 6B shows similar data for DL for $\sigma = 3.75$ min. and separations of 7.5 and 37.5 min (2 and 10 times $\sigma$). Figure 7 shows bisection thresholds vs contrast for observer CN, for $\sigma = 1.875$ min, and separations of 3.75 and 18.75 min (2 and 10 times $\sigma$). Her data are similar to those of DL in showing a stronger contrast dependence for same-polarity lines.
with small separations than for either large separations or opposite-polarity lines; however, her data at $S = 2 \sigma$ show a somewhat shallower slope than DL's data.

The data of DL from Fig. 6A, B are combined and replotted in Fig. 8, with the ordinate specified in standard deviation units. Here it can be seen clearly that at both scales, when the separation is $2 \sigma$, thresholds for same-polarity stimuli improve approximately in proportion to contrast over the whole range tested. This is the reason thresholds get as low as $0.02 \sigma$. On the other hand opposite-polarity stimuli of the same separation show much less contrast dependence, similar to that observed with same-polarity lines which have a larger separation ($S \approx 5 \sigma$).

**DISCUSSION**

The present experiments with Gaussian blurred lines show that over a wide range of stimulus blurs, when the separation is small with respect to the stimulus blur (between about 2 and $5 \sigma$), spatial position discrimination is much better for same- than for opposite-polarity stimuli. When the separation is large with respect to the blur ($> 5 \sigma$), thresholds are independent of stimulus polarity. At very small separations ($\approx \sigma$), the situation reverses so that thresholds for opposite-polarity lines are better than thresholds for same-polarity lines. The reader can confirm many of these observations by inspecting Fig. 1 (examples of our stimuli), and Fig. 9, which shows the luminance profiles corresponding to the stimuli of Fig. 1. The bottom four luminance profiles are for same-polarity Gaussian lines, with separations (from top to bottom) equal to 1, 2, 3 and 10 $\sigma$. The top four luminance profiles are opposite-polarity lines (note that the polarity is inverted from Fig. 1). In each of the profiles the middle line has been offset by a constant amount, $0.2 \sigma$, just as in Fig. 1. At the largest separation (10 $\sigma$), the offset is equal to $\frac{1}{2}$ of the separation, similar to the thresholds of our observers, independent of polarity. At separations of 3 and 2 $\sigma$, the offsets are very visible for the same-polarity profiles; however, they are almost invisible in the opposite-polarity profiles. At a separation of $\sigma$, the asymmetry in the opposite-polarity profile
FIG. 6. Spatial interval discrimination thresholds as a function of contrast (the abscissa specifies the mean contrast; however, from trial to trial, the contrast varied randomly by 30% of the nominal value). (A) shows data for DL for $\sigma = 1'$ min. The small and large solid circles are for dark lines with separations equal to 2 and 10 min respectively. Note that for dark lines with separation equal to 2 min, contrast has a strong effect upon thresholds. For separation of 10 min the contrast dependence is much less. The open squares show the effects of contrast on opposite-polarity stimuli with a separation of 2 min. The solid line shows a slope of $-1$ (i.e. threshold is proportional to contrast). (B) shows similar data for DL for $\sigma = 3.75'$ min, and separations of 7.5 and 37.5 min (2 and 10 times $\sigma$).

FIG. 7. Similar to Fig. 6, for observer CN for $\sigma = 1.875'$ min, and separations of 3.75 and 18.75 min (2 and 10 times $\sigma$).

FIG. 8. The data of Fig. 6A, B for DL are combined and replotted, with the ordinate specified in standard deviation units. Here it can be seen clearly that at both scales, when the separation is 20, thresholds for same-polarity stimuli improve approximately in proportion to contrast over the whole range tested. On the other hand opposite-polarity stimuli of the same separation show much less contrast dependence, similar to that observed with same-polarity lines which have a larger separation ($S \approx 5 \sigma$).

due to the offset is quite apparent, and this asymmetry corresponds to the contrast cue that the observers report using. No such asymmetry occurs for the same-polarity profile at separation equal to $\sigma$, and the offset is essentially invisible. Our results confirm and extend the previous studies of Levi and Westheimer (1987), Morgan (personal communication) and Murphy et al. (1988). Our results also show clearly why Toet (1987) failed to observe differences in Vernier alignment with low contrast blobs of same- and opposite-polarity. Toet used a large separation of about 14 $\sigma$, where, according to the present results, same- and opposite-polarity stimuli give approximately equal thresholds. The present study also shows that at small separations (e.g. $S = 2 \sigma$), spatial interval thresholds for same-polarity lines are strongly contrast dependent, with threshold almost inversely proportional to contrast. However, at large separations (e.g. $S = 10 \sigma$), thresholds for same-polarity stimuli show little contrast dependence. Interestingly, for opposite-polarity lines, neither small nor large separations result in a strong contrast dependence.

Based upon their experiments with thin lines, Levi and Westheimer (1987) suggested that within a "spatial processing module" of approx. 4 min, similar in size to a foveal ocular dominance column (Levi et al., 1985), the visual system is unable to make high-fidelity position comparisons between stimuli of opposite
Spatial interval discrimination

3-Line profiles with offset = 0.2 \sigma

Fig. 9. Luminance profiles corresponding to the stimuli of Fig. 1. The four luminance profiles at the top of Fig. 9 are for opposite-polarity Gaussian lines, with separations (from top to bottom) equal to 1, 2, 3 and 10 \sigma (note that the polarity is inverted from Fig. 1). The bottom four luminance profiles are for same-polarity lines. In each of the profiles the middle line has been offset by a constant amount, 0.2 \sigma. At the largest separation (10 \sigma), the offset is similar in magnitude to the thresholds of our observers, independent of polarity. At separations of 2 and 3 \sigma, the offsets are very visible for the same-polarity profiles, however, they are almost invisible in the opposite-polarity profiles. At separation equal to \sigma, the asymmetry in the opposite-polarity profile due to the offset is quite apparent, while it is essentially invisible in the same-polarity profile at separation equal to \sigma.

Fig. 9. Luminance profiles corresponding to the stimuli of Fig. 1. The four luminance profiles at the top of Fig. 9 are for opposite-polarity Gaussian lines, with separations (from top to bottom) equal to 1, 2, 3 and 10 \sigma (note that the polarity is inverted from Fig. 1). The bottom four luminance profiles are for same-polarity lines. In each of the profiles the middle line has been offset by a constant amount, 0.2 \sigma. At the largest separation (10 \sigma), the offset is similar in magnitude to the thresholds of our observers, independent of polarity. At separations of 2 and 3 \sigma, the offsets are very visible for the same-polarity profiles, however, they are almost invisible in the opposite-polarity profiles. At separation equal to \sigma, the asymmetry in the opposite-polarity profile due to the offset is quite apparent, while it is essentially invisible in the same-polarity profile at separation equal to \sigma.

polarities. The present results suggest that there may be similar spatial processing modules at many spatial scales, and that the approx. 4 min modules evident in many previous experiments using thin lines as stimuli (e.g. Westheimer & Hauske, 1975; Westheimer & McKee, 1977a; Levi et al., 1985), represent the smallest modules.

How can one account for the large loss in precise position acuity with closely spaced opposite-polarity lines (S < 5 \sigma)? Levi and Westheimer (1987) conjectured that such a loss of precision could be a consequence of independent processing of the stimulus features (consisting of an increment and an adjacent decrement) by on- and off-center retinal ganglion cells. For example, the on- and off-center retinal ganglion cells occupy different layers in the retina (Nelson, Famiglietti & Kolb, 1978) and each forms a regular mosaic (Wassle, Peichl & Boycott, 1983). The projections of on- and off-center cells remain separate at the lateral geniculate nucleus (Schiller, 1984), and there is evidence that selective abolition of the on-pathway produces selection losses in the detection of increments (Schiller, 1986). If precise spatial localization of closely spaced features depends upon a comparison process (Westheimer, 1975), then it might be expected that comparisons of the positions of opposite-polarity features that are processed separately and independently would be more difficult than comparisons of similar features. If this explanation is correct, then it implies a qualitatively different process operates at large separations (S > 5 min), where thresholds are independent of polarity. The present results that it is the separation in standard deviation units rather than in minutes that is critical, makes the on-off segregation explanation less appealing.

Computational models for position acuity

There are now several models of spatial vision which account for the high precision of spatial interval discrimination for closely spaced features on the basis of localized spatial filters (Klein & Levi, 1985; Carlson & Klopfenstein, 1985; Wilson, 1986). Thus, it is reasonable to apply these models to opposite-polarity targets. We performed viewprint computations (Klein & Levi, 1985) for 2-line spatial interval discrimination with same- and opposite-polarity thin lines. The viewprint computation is based upon the differential responses of localized spatial mechanisms of different sizes and positions (Klein & Levi, 1985). Figure 10 shows the predicted thresholds over a range of separations. For same-polarity lines: (1) the optimal threshold is about 2 sec and occurs at a separation between 1 and 2 min, about 2–3 times the

![Fig. 10. Shows threshold predictions of the "Viewprint" model (Klein & Levi, 1985), for 2-line spatial interval discrimination as a function of separation, for same- and opposite-polarity stimuli.](image-url)
intrinsic blur of the visual system (Levi & Klein, 1990a); (2) for separations smaller than the optimum, thresholds are sharply degraded. For separations larger than the optimum, they obey Weber's law with a Weber fraction of about 0.02. For opposite-polarity lines: (1) at separations greater than about 4 min, thresholds are approximately equal to those for same-polarity lines; (2) for separations between 1 and 4 min, thresholds are markedly degraded; and (3) at separations <1 min, thresholds are actually better than those for same-polarity lines. Many of the features of our data are evident in the viewprint predictions. The strong contrast dependence of same-polarity 3-line interval discrimination at a separation of 2 σ is also consistent with the viewprint model, since the differential response is based upon the detection of a luminance cue. A shift of the middle line of the 3-line stimulus produces a change in contrast in the gaps between the centers of the lines which is proportional to the contrast times the shift. At the detection threshold the change in contrast (ΔC) is a constant, so the threshold shift is inversely proportional to contrast, in agreement with the data. For the same-polarity stimuli, the high sensitivity is achieved because the plateau in the luminance profile (see Fig. 9) allows a comparison of the luminances. In fact, Klein and Levi (1985) achieved their record-breaking hyperacuity thresholds by placing flanking lines around their 3-line bisection stimulus, to extend this luminance plateau. The lack of contrast dependence of the opposite-polarity thresholds suggests that this cue cannot be used for opposite-polarity stimuli.

It is both informative and surprising to look at the viewprint predictions in more detail. The reason that the viewprint model predicts degraded performance for closely spaced opposite-polarity stimuli, is that the largest differential response to these stimuli is based on the outputs of smaller rather than larger mechanisms as compared to same-polarity stimuli with the same separation. The null point cue for same-polarity lines occurs when adjacent stimulus lines are separated by a half cycle of the mechanism profile. For opposite-polarity lines the null point cue occurs when adjacent stimulus lines are separated by a full cycle of the mechanism profile. The very high spatial frequency of these smaller filters causes them to have a lower signal-to-noise ratio, and therefore poor position sensitivity. Although we have not performed viewprint calculations with Gaussian blurred lines, a multiple size-tuned mechanism model such as the viewprint model could, in principle, accommodate the results with blurred lines; however, it is not clear whether the viewprint model will make similar predictions for large, blurred lines, without a second stage filtering operation, since the contrast sensitivity of the filters may not be a critical limiting factor for blurred lines.

It is worth noting that the viewprint approach is not the only model that predicts either hyperacuity thresholds, or differential sensitivity for same- and opposite-polarity stimuli. For example, a simple intensity-based (∆I/I) model (Hartridge, 1922, Morgan & Aiba, 1985) is able to predict thresholds of 0.02 σ. It also predicts worse thresholds for opposite- than for same-polarity lines. However, models based on local intensity differences also predict higher sensitivity to blurred dark than to blurred light lines, since the denominator of ∆I/I is lower for blurred dark lines. While two of our observers did in fact show slightly better thresholds for dark than for light lines with a 1 min standard deviation (see Fig. 2, and also Watt & Morgan, 1983), our data show little or no difference in thresholds for dark and light lines at larger standard deviations where a ∆I/I model would be expected to perform best.

An alternative approach is based upon the contrast Weber fraction (∆C/C). The ΔC in the numerator is the maximum difference in contrast between the two patterns to be discriminated. The C in the denominator is the maximum local contrast (not necessarily at the same point where ΔC is maximum). This approach, which is described in detail elsewhere (Klein & Levi, 1989), provides an excellent account of 2-line resolution and 2-line spatial interval discrimination with blurred lines (Levi & Klein, 1990a). This Weber's law model also predicts many qualitative aspects of the present data; however, it is not sufficiently sensitive to account for the very low 3-line spatial interval thresholds obtained with same-polarity lines.

Another approach to the problem is to ask what information would be available to a localized center-surround receptive field, such as a retinal ganglion cell. To test this, we took the 2nd derivative of our Gaussian stimuli, and then solved for a local contrast change in the pattern (maximum change/peak response) equal to 0.1, the human contrast Weber fraction (Badcock, 1984). The results, like the data, show that, for same-polarity lines, thresholds are optimal at a
separation of around 2 \( \sigma \), and have a threshold of about 0.02 \( \sigma \). At both larger and smaller separations, thresholds are degraded. However, the second derivative model does too well for opposite-polarity lines. It predicts opposite-polarity thresholds of about 0.04 \( \sigma \), considerably better than that obtained by our observers.

**SUMMARY AND CONCLUSIONS**

Our results show that spatial interval discrimination thresholds are essentially scale invariant. Over a wide range of stimulus blurs, the thresholds for same-polarity stimuli are optimal (\( \approx 0.02 \sigma \)) at a separation of about 2 \( \sigma \), and are degraded at smaller separations. For thin, unblurred lines, performance will be limited by the intrinsic blur of the visual system which has a standard deviation of about 0.5 min (Levi & Klein, 1990a). Stimuli with opposite-polarity lines have a different separation function than the same-polarity data (more or less inverted for separations less about 5 \( \sigma \)), so that at separations between 2 and about 4 \( \sigma \), thresholds are strongly degraded. This degradation is predicted qualitatively (and with differing degrees of success quantitatively) by several models of spatial vision.

While we generally consider hyperacuity in terms of the very tiny spatial thresholds which obtain at small separations with stimuli comprised of thin lines, the present results, and those of others (Levi & Klein, 1990a; Toet et al., 1987) suggest that for same-polarity stimuli, hyperacuity thresholds are a general property of the visual system, which occur at many spatial scales.

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**REFERENCES**


