EQUIVALENT INTRINSIC BLUR IN SPATIAL VISION

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Abstract—We used Gaussian blurred stimuli to explore the effect of blur on three tasks: (i) 2-line “resolution”; (ii) line detection; and (iii) spatial interval discrimination, in both central and peripheral vision. The results of our experiments can be summarized as follows.

(i) 2-Line “resolution”: thresholds for pairs of unblurred, low contrast, stimuli are approx. 0.5 min arc in the fovea. When the stimulus blur is small, it has little effect upon 2-line “resolution”; however, when the stimulus blur, σ, exceeds 0.5 min, thresholds are degraded. We operationally define this transition point as the equivalent intrinsic blur or \( B_e \). When the standard deviation of the stimulus blur, σ, is greater than \( B_e \), then the “resolution” threshold is approximately equal to σ. Both the unblurred “resolution” threshold, and the equivalent intrinsic blur, \( B_e \), vary with eccentricity in a manner consistent with the variation of cone separation within the central 10 deg. When the stimulus blur exceeds the equivalent intrinsic blur, “resolution” in the periphery is the same as in the fovea.

(ii) Line detection: when the standard deviation of the stimulus blur, σ, is less than \( B_e \), then the line detection threshold is approximately inversely proportional to σ (it is \( T_{det} \sim 1/\sigma \)) i.e. it obeys Riccio’s law. When the standard deviation of the stimulus blur, σ, is greater than \( B_e \), then the “resolution” threshold is approximately equal to σ and the detection threshold is approximately a fixed contrast (to be referred to as \( T_{det} \)).

According to (i) and (ii), the equivalent intrinsic blur, \( B_e \), plays a dual role in determining both the “resolution” threshold and the detection threshold, \( B_e \) corresponds to the “Ricci’s diameter” for spatial summation in a detection task, and it also corresponds to the “resolution” threshold for thin lines. This connection between detection and “resolution” is somewhat surprising.

(iii) Spatial interval discrimination: thresholds are proportional to the separation of the lines (i.e. Weber’s law). At the optimal separation, the thresholds represent a “hyperacuity” (i.e. they are smaller than the “resolution” threshold). For unblurred lines, the optimal separation is approximately 2–3 times the “resolution” limit at all eccentricities, so the optimal separation varies with eccentricity at the same rate as the equivalent intrinsic blur, \( B_e \). However, the optimal spatial interval threshold falls off with eccentricity about 3–4 times more rapidly, consistent with the rate of decline of other position acuity tasks. For Gaussian blurred lines, over a wide range of separations and eccentricities, spatial interval discrimination thresholds begin to rise when the stimulus blur exceeds between about \( f \) and \( 2 \) the separation of the lines. The strong elevation of the optimal spatial interval discrimination threshold in the periphery cannot be predicted on the basis of detectability of the lines, “resolution”, or on the basis of the equivalent intrinsic blur. We hypothesize that the increased spatial interval discrimination thresholds are a consequence of position uncertainty, perhaps due to sparse spatial sampling in the periphery.

Resolution Spatial interval discrimination Hyperacuity Blur Gaussian blur Intrinsic blur Spatial vision

INTRODUCTION

Human visual performance may be limited by both external and intrinsic factors. For example the detection threshold of a spot of light may be constrained by quantum fluctuations in the stimulus, or intrinsic uncertainty in the visual system of the observer (Barlow, 1956; Cohn & Wardlaw, 1985). One approach to studying the factors which limit normal visual performance, is to attempt to estimate the intrinsic limitations to performance. An example of this approach is Barlow’s estimate of his observers’ “dark noise”, i.e. neural noise in the visual system, whose effect was equivalent to that of a real stimulus (Barlow, 1956). A more recent example is Pelli’s (1984) estimate of the equivalent intrinsic noise limiting contrast detection in the human visual system.

In spatial vision, one critical limiting factor is blur, or the spread of the image. Blur may be a property of the visual stimulus, the optics of the eye, or of the visual nervous system. A large number of experiments have examined
the effect of stimulus blur upon performance. For example, the effects of stimulus blur upon detection (Shapley, 1974; Legge & Kersten, 1983), resolution (Hartridge, 1923; Toet & Koenderink, 1987), Vernier acuity (Stigmar, 1971; Watt, Morgan & Ward, 1983; Williams, Enoch & Essock, 1984; Toet, van Eekhout, Simons & Koenderink, 1987; Toet, Simons & Koenderink, 1988; Watt & Hess, 1987), bisection acuity (Toet et al., 1987), spatial interval discrimination (Watt & Morgan, 1983) and stereo-acuity (Stigmar, 1971; Westheimer & McKee, 1980) have all been examined. Stimulus blur affects performance on each of these tasks differently. Moreover, on a given task, such as Vernier acuity, the effect of stimulus blur depends upon the target configuration (Stigmar, 1971; Williams et al., 1984).

In the present paper we used Gaussian blurred stimuli to explore the effect of blur on three 1-dimensional spatial tasks: (i) 2-line "resolution"; (ii) line detection; and (iii) 2-line spatial interval discrimination, in both central and peripheral vision. For our 2-line task, by varying the separation between the lines, classical 2-line "resolution" acuity, and spatial interval discrimination, a "hyperacuity" (Westheimer, 1975), blend smoothly; moreover, these stimuli are relatively simple to model.

Specifically, in the first part of the paper we measure the effect of Gaussian blur on 2-line "resolution". The results at each eccentricity can be summarized by the following equation:

\[ Th = k \sqrt{\sigma^2 + B_i^2}; \]  

where \( \sigma \) is the stimulus blur, specified as the standard deviation of the Gaussian, \( B_i \) is the equivalent intrinsic blur, and \( Th \) is the resolution threshold. This is the formulation proposed by Watt and Morgan (1984), and later used by Watt and Hess (1987), to estimate the "intrinsic blur" in a Vernier acuity task. This model is based on the assumption that the visual system has an additive internal error that acts like blur (\( B_i \)) and a multiplicative factor, \( k \), that depends on such things as the point on the psychometric function at which threshold is defined. The variance of the stimulus blur is expected to add to the variance of the intrinsic blur since the two are uncorrelated. Thus \( B_i \) is operationally defined as the amount of Gaussian blur that that raises threshold by 40% (from \( kB_i \) to \( \sqrt{2} kB_i \)). In the fovea, 2-line "resolution thresholds" for unblurred stimuli and \( B \), are about 0.5 min since \( k \) is found to be close to unity. When \( \sigma \) exceeds \( B_i \), "resolution" thresholds are approximately equal to \( \sigma \). We argue that the resolution task is appropriate for estimating the limiting equivalent intrinsic blur of the visual system. Our results suggest that this equivalent intrinsic blur is independent of stimulus contrast, and that it reflects the optical quality of the retinal image and discrete sampling by the cones (within the central 10 deg).

Since optical blur is not the only factor contributing to the intrinsic blur, the term equivalent intrinsic blur will be used. The second part of this paper shows that equivalent intrinsic blur has important consequences for detection of a line, and for spatial interval discrimination.

**METHODS AND PROCEDURES**

**Stimuli**

The stimuli in each of the experiments were horizontal dark Gaussian blurred lines. Line stimuli are spatially localized, and can easily be subjected to controlled blur, so that the spread of the line and its contrast can be varied independently. Figure 1 shows examples of five Gaussian blurred line pairs, each with a different separation (from top to bottom 0, 1, 2, 2.79 and 3.5 SD). In the experiments, the Gaussian was truncated at \( \pm 4 \) SD. In particular, we are interested in the effect of the blur spread, specified as the standard deviation of the Gaussian. The stimuli were generated by a programmable arbitrary function generator under computer control, and were presented on a Joyce CRT with a mean luminance of 115 cd/m². The function generator was synchronized to the fast sweep of the Joyce, and two programmable attenuators (one linear and the other logarithmic) in series provided excellent control of stimulus contrast and timing.

For foveal viewing, the lines were approx. 1 deg long. For peripheral viewing the stimuli were presented in the lower visual field and the stimulus dimensions were scaled by varying the viewing distance according to \( D = d_f/(1 + E/2.5) \), where \( E \) is the eccentricity in degrees, and \( d_f \) is the distance used for foveal viewing (5.5 m). Scaling the stimulus size had two main effects. (1) The lines got longer with increasing eccentricity. The 1 deg line length for foveal viewing is considerably longer than necessary for asymptotic performance (Westheimer & McKee, 1977; Klein & Levi, 1985), thus increasing the stimulus length in the periphery, ensured that the stimuli were
Fig. 1. Examples of the stimuli used in these studies. Shown here are four pairs of Gaussian blurred lines.
sufficiently long to provide adequate sampling (Levi & Klein, 1986) at all eccentricities. Control experiments verified that further increases in the length of the lines had no effect upon thresholds. (2) The lines got wider with increasing eccentricity. For thin lines (less than the eye’s line spread function) on a uniform background, visibility is a function of the product of contrast and width. Thus reducing the distance with increasing eccentricity enabled us to use approximately the same physical contrasts at each eccentricity, and maintain equal visibility.

Viewing was monocular (to allow comparison with the results of amblyopic eyes presented in the subsequent paper), and with natural pupils. In order to minimize temporal transients, the stimuli were ramped on over 300 msec, remained at a plateau for 600 msec, and were ramped off over 300 msec.

Stimulus luminance and contrast (ΔL/L\text{mean}), were calibrated at regular intervals throughout the duration of these experiments, using a Pritchard SpectraPhotometer. Contrast values reported here refer to the peak contrast of the Gaussian stimuli.

**Experiment I: 2-line resolution**

Unless otherwise specified, the line contrast was 5 times threshold (determined in expt II) for all eccentricities and blurs; however, in order to minimize contrast cues to resolution or separation, we introduced a contrast jitter of 30% from trial to trial. Absolute position cues were eliminated by jittering the position of the line pair.

Thresholds for both 2-line resolution and spatial interval discrimination were measured using a self-paced rating-scale method of constant stimuli. The resolution task essentially measures the smallest separation between the lines that can be discerned (i.e. the smallest separation that can be reliably distinguished from no separation). For the resolution experiments, the line-pair was either overlapping (i.e. no separation), or was vertically separated by a distance approximately equal to ½, 1, 1½ or 2 times the resolution limit (as estimated from pilot experiments) at each eccentricity and blur level. In an experimental run of 125 trials, preceded by 10–30 practice trials, the observer’s task was to judge the separation of the lines by giving numbers from 0 (overlapping) to 4. Resolution thresholds were obtained by calculating a maximum-likelihood estimate of the d' values for each stimulus and constructing a psychometric function relating d' to stimulus offset. In pilot experiments using large blocks of trials (250–1000 trials per run), we allowed the exponent of the psychometric function to float. For a number of observers, conditions (normal and amblyopic), and eccentricities, we found the exponent to be between about 1.2 and 1.9, with a mean of 1.5. The exponent did not vary systematically with eccentricity or condition; thus, the resolution thresholds reported here were calculated by fixing the exponent at 1.5, and interpolating to a d' = 1. The thresholds reported are the mean of 2–5 runs, weighted by the inverse variance, and the error bars reflect both within and between run variance. There might be objections to our use of the term “resolution” for our task in which width and blur discrimination cues are available (see Results). However, we should point out that our stimuli are similar to those used in classical studies of 2-line resolution (Helmoltz, 1909; Wilcox & Purdy, 1933; see also Geisler, 1989), and the task is one of distinguishing a single line from a double line. To distinguish our task from resolution tasks which do not have width cues, we will henceforth use quotation marks around the word.

**Experiment II: detection of Gaussian blurred lines**

Contrast thresholds for detecting single Gaussian blurred lines as a function of Gaussian spread and eccentricity, were measured using a self-paced rating-scale method of constant stimuli. Specifically, a detection run consisted of 100 trials. In each trial, marked by a tone, the stimulus was either a “blank” (zero contrast), or one of three closely spaced contrast levels chosen to span the threshold (estimated in preliminary studies). The observer’s task was to rate the visibility of the stimulus, by giving numbers from 0 (blank) to 3. The computer randomized the order of presentation, tallied the observer’s responses, and provided feedback following each trial. Thresholds for detection were obtained by calculating a maximum likelihood estimate of the d' values of each stimulus and constructing a psychometric function with an exponent of 2, relating d' to stimulus contrast (Klein & Stromeyer, 1980; Legge, 1984). The contrast threshold was defined by interpolation to a d' = 1. The thresholds reported are the mean of 2–5 runs, weighted by the inverse variance, and the error bars are ±1 standard error, reflecting both within and between run variance (Klein & Levi, 1987).
Experiment III: 2-line spatial interval discrimination

The spatial interval discrimination task measures the ability to judge the extent of the separation between a pair of lines for a range of base separations. In the present study spatial interval discrimination thresholds were measured for a wide range of base separations and stimulus blurs (at each eccentricity). The stimuli and methods were essentially identical to those used to measure resolution thresholds. In an experimental run the line-pair was presented with one of five randomly chosen closely spaced vertical separations, and the observer’s task was to judge whether the separation between the lines was wider or narrower than the implicit reference (i.e. the mean of the five separations) by giving numbers from -2 to +2. This method is similar to that used by Westheimer and McKee (1977) for 2-line spatial interval discrimination. Thresholds for spatial interval discrimination were obtained with the exponent of the psychometric function equal to 1 (i.e. d' is proportional to separation). This is a multiple-criterion probit analysis.

Four highly practiced observers participated in one or more of the experiments. Observers DL and HD had normal binocular vision and corrected-to-normal visual acuity in each eye. DL (an author) participated in all experiments. We also extensively tested the nonamblyopic eyes of two amblyopic observers. WP is a strabismic amblyope, KP is an anisometropic amblyope. Both have corrected-to-normal visual acuity in their nonamblyopic eyes. Details of the visual characteristics of all of the observers are provided in Table 1 of the following paper (Levi & Klein, 1990a). All testing was monocular, and the observers were all carefully refracted for the experiments.

Experiment IV: 2-line resolution and spatial interval discrimination with unblurred (thin) lines

For these experiments the stimuli were thin, horizontal, rectangular, dark lines presented on a uniform background (mean luminance 115 cd/m²). For foveal viewing the lines were approx. 1 deg long, and 0.14 min wide. For peripheral viewing they were scaled in size by varying the viewing distance as described above. Unless otherwise stated, the line contrast was approx. 5 times the detection threshold (with a jitter of 30%). All other experimental details were identical to those described above.

RESULTS AND DISCUSSION

Experiment I: effects of Gaussian blur on resolution

Figure 2A and B shows 2-line “resolution” thresholds in minutes plotted as a function of the standard deviation of the Gaussian blur. The “resolution” threshold represents the smallest line separation which can be reliably distinguished from a pair of lines with zero separation, and the figure shows the data of two highly practiced observers at 0 (O), 2.5 (∆), 5 (□) and 10 deg (●) in the lower visual field. The lines fit to the data of Fig. 2 are of the form shown in equation (1) which can be rewritten as:

\[ T_h = T, \left[1 + \left(\frac{\sigma}{B_i}\right)^2\right]^{1/2}, \]  

where \( T_h \) is the “resolution” threshold for an unblurred stimulus (i.e. the asymptotic threshold value which equals \( k B_i \) from equation 1), \( \sigma \) is the stimulus blur, and \( B_i \) is the equivalent intrinsic blur (i.e. the horizontal position of the knee in the function). This model is based on the assumption that the visual system has an internal error that acts like blur \( (B_i) \). When the stimulus blur is small, it has little influence upon “resolution” thresholds; however, when it exceeds \( B_i \), then threshold is proportional to the stimulus blur. Thus the parameter \( B_i \) represents an estimate of the intrinsic error in the visual system. Below we will argue that \( B_i \) represents the limitations imposed by the eye’s optics, and by discrete spatial sampling by cones within the central 10 deg.

The results at each eccentricity, given by equations (1) or (2) have the following properties:

(1) for small amounts of blur, there is an asymptotic region where stimulus blur has little effect upon the threshold \( (T_h) \);

(2) there is an intermediate blur level, below which stimulus blur has little effect, and above which thresholds rise in proportion to the stimulus blur. We operationally define the amount of Gaussian blur, \( \sigma \), that raises threshold by 40%, as the equivalent intrinsic blur \( (B_i) \). As will be demonstrated below, \( B_i \) reflects constraints imposed by both optical and neural blur, but is contrast independent. In the fovea, both the unblurred “resolution” threshold \( (T_h) \) and the equivalent intrinsic blur \( (B_i) \) are approx. 0.4-0.7 min (see Fig. 13 for data showing resolution thresholds of 0.35 min). This range of values is consistent with both the eye’s blur function, and the spacing of foveal cones.
Equivalent intrinsic blur

within the central 10 deg. With increasing eccentricity, both $T_c$ and $B_i$ increase systematically, although the eye's optics have changed hardly at all:

(3) when the stimulus blur exceeds the equivalent intrinsic blur, "resolution" thresholds rise in proportion to the stimulus blur. More specifically, we find that in both central and peripheral vision, for blur values $> B_i$, "resolution" is approximately equal to the standard deviation of the blur (actually 0.9 times the standard deviation). Thus the constant $k$ in equation (1) is 0.9, independent of eccentricity. Toet et al. (1987) also found proportionality between the 2-blob "resolution" threshold, and the standard deviation of their Gaussian blobs at low contrast levels. Their proportionality constant of $k = 1.5$ is slightly larger than ours, presumably because their stimuli were at very low contrast levels.

At each eccentricity, the model provides a reasonable fit to our data, and the nonlinear regression using equation (2) provides estimates of the parameters and their error bars. Both $T_c$ and $B_i$ increase approximately linearly with eccentricity. This approximately linear variation in visual performance, or in the anatomical or physiological functions which may constrain performance with eccentricity (Weymouth, 1958), can be simply summarized

Fig. 2. 2-Line "resolution" thresholds (in minutes) plotted as a function of the standard deviation of the Gaussian blur (in minutes). The "resolution" threshold represents the smallest line separation which can be reliably distinguished from a pair of overlapped lines, and the figure shows the data of DL (A) and WP (B), at 0 (○), 2.5 (△), 5 (□) and 10 deg (●) in the lower visual field. The lines fit to the data are of the form:

$$Th = T_c[1 + (\sigma/B_i)^2]^{1/2},$$

where $T_c$ is the "resolution" threshold for an unblurred stimulus (i.e. the asymptotic threshold value), $\sigma$ is the stimulus blur, and $B_i$ is the equivalent intrinsic blur for "resolution" (i.e. the knee in the function). Note that the asymptotic thresholds and the equivalent intrinsic blur (the corner blur) increase systematically with eccentricity. At large Gaussian blur spreads, "resolution" is independent of eccentricity, but is proportional to the Gaussian standard deviation. (C) The data of DL from (A) are replotted with both the "resolution" thresholds, and the Gaussian standard deviation specified as a fraction of eccentricity $E + E_i$ (where $E_i = 2.1$ deg). In these units, both the asymptotic thresholds and the equivalent intrinsic blur are about 0.5 min $+ (1 + E/2.1$ deg $) = 0.0035 \ (E + 2.1$ deg $)$ at each eccentricity. For stimulus blur values exceeding the equivalent intrinsic blur, thresholds are approximately equal to the standard deviation of the stimulus blur.
by a single dimensionless parameter, which we call $E_2$, i.e. the eccentricity at which the foveal threshold doubles. Table 1 summarizes the fall-off of both $T_1$ and $B_i$ (as well as many other functions) with eccentricity. For both of these functions the value of $E_2$ was approx. 2.1 deg. The absolute values of $T_1$ and $B_i$ are given in Table 2 of the following manuscript (Levi & Klein, 1990a). It is worth noting that neither the Gaussian data shown here, nor the detection or cone spacing data fall-off precisely linearly (see Fig. 14; also see Curcio, Sloan, Packer, Hendrickson & Kalina, 1987; Levi, Klein & Aitsebaomo, 1985). Thus, the value of $E_2$ will vary slightly with eccentricity; however, over the limited range of eccentricities tested, the linear fits to the data provide a reasonable approximation, and the $E_2$ values give a reasonable estimate of the rate of variation with eccentricity. Figure 2C shows the “resolution” thresholds of DL from each eccentricity replotted as a function of the stimulus blur; however, now both the thresholds, and the Gaussian standard deviation have been replotted as a fraction of “effective eccentricity”; $E + E_2$ (2.1 deg or 126 min). In these units, both the unblurred thresholds and the equivalent intrinsic blur are about 0.0035 ($E + 126$ min), approximately equal to cone spacing at each eccentricity.

**Effect of contrast on “resolution”**. Figure 3A shows “resolution” thresholds as a function of the spread of the stimulus. For each curve shown, the lines had a different contrast, either 1.25 (○), 2.5 (□), or 10 (△) times threshold. Note that when the pair of lines are close together (within Ricco’s area) they will summate and will be 2.5, 5.0 or 20 times threshold respectively. The curves fit to each data set are more or less parallel. This suggests that there is a range of contrast values over which “resolution” thresholds depends upon contrast (note that WP’s data at 5 times threshold (Fig. 2B) is slightly higher than the 10 times threshold data); however, the knee in each of the curves occurs at approximately the same place, suggesting that the equivalent intrinsic blur is more or less independent of stimulus contrast. Contrast has an effect only on the multiplicative factor, $k$, such that at very low contrasts, thresholds are degraded at all blur values, so the curves are elevated uniformly. As contrast increases, the “resolution” thresholds gradually decrease so that at high contrast the thresholds can be as low as 0.33’ (Carney, Klein & Levi, 1988). The constancy of $B_i$ can be seen more clearly in Fig. 3B, where the equivalent intrinsic blur estimated from the fit to the data is plotted against stimulus contrast (expressed in threshold units). The data of WP from Figs 3A and 2B are shown by (○). Also shown are “resolution” (●) data from another observer (KP). The results show clearly that the equivalent intrinsic blur is essentially contrast invariant.

Figure 4 plots the equivalent intrinsic blur ($B_i$) against the unblurred “resolution” threshold ($T_1$) for central and peripheral vision. The data are from Fig. 2 for contrasts 5 times the detection threshold. Also shown in this figure are the effects of optical defocus (●), which are similar to the effects of Gaussian stimulus blur. For each observer and condition, $T_1$ and $B_i$ are approximately equal. To be more precise, $B_i = 1.1 T_1$. 

### Table 1. $E_2$ values

<table>
<thead>
<tr>
<th>Task</th>
<th>Parameter</th>
<th>Observer</th>
<th>Figure</th>
<th>$E_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Detection</td>
<td>Visibility of a thin line</td>
<td>HD</td>
<td>○</td>
<td>2.7 ± 0.3</td>
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<tr>
<td></td>
<td>(threshold in % minutes)</td>
<td>DL</td>
<td>□</td>
<td>2.0 ± 0.4</td>
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<td>&quot;Resolution&quot;</td>
<td>Size of $B_i$ (critical Gaussian standard deviation for detection)</td>
<td>DL</td>
<td>○</td>
<td>2.4 ± 0.2</td>
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<tr>
<td></td>
<td>Equivalent intrinsic blur ($B_i$)</td>
<td>DL</td>
<td>□</td>
<td>3.5 ± 0.3</td>
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<tr>
<td></td>
<td>Asymptotic &quot;resolution&quot;</td>
<td>DL</td>
<td>○</td>
<td>2.1 ± 0.3</td>
</tr>
<tr>
<td></td>
<td>thresholds ($T_1$)</td>
<td>DL</td>
<td>□</td>
<td>2.4 ± 0.3</td>
</tr>
<tr>
<td></td>
<td>2-Line &quot;resolution&quot;</td>
<td>HD</td>
<td>○</td>
<td>2.1 ± 0.3</td>
</tr>
<tr>
<td></td>
<td>thresholds (thin rectangular black lines)</td>
<td>DL</td>
<td>□</td>
<td>2.5 ± 0.3</td>
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<tr>
<td>Spatial interval</td>
<td>Optimal thresholds</td>
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<td>○</td>
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<td>discrimination</td>
<td>(2 thin black lines)</td>
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<td>□</td>
<td>0.82 ± 0.1</td>
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<td>WP</td>
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<td></td>
<td></td>
<td>WP</td>
<td>□</td>
<td>0.53 ± 0.1</td>
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Equivalent intrinsic blur

For each curve shown, the lines had a different contrast, either 1.25 (○), 2.5 (□), or 10 (△) times threshold. The curves are the model fits to the data. (B) The equivalent intrinsic blur $B_i$, estimated from the model fit to the data, is plotted against stimulus contrast (expressed in threshold units). The data of WP from (A) are shown by the open circles. Also shown are “resolution” data (○) from another observer (KP). The results show that the equivalent intrinsic blur, $B_i$, is essentially contrast invariant.

To summarize, we can say that when the external stimulus blur is less than the equivalent intrinsic blur, then the “resolution” threshold is equal to 0.9 times the intrinsic blur. When the external blur is greater than the equivalent intrinsic blur, then the “resolution” threshold is equal to 0.9 times the standard deviation of the externally imposed stimulus blur. This in turn suggests that 2-line “resolution” thresholds at all levels of blur reflect a general processing strategy of the visual system rather than simply the limitations imposed by cones or ganglion cells.

Figure 5 illustrates pairs of Gaussians separated by 0, 1, 2 and 2.79 and 3.5 SD. Note that these profiles correspond to the stimuli shown in Fig. 1. The “textbook” picture of “resolution” is that two lines can be resolved when there is just-noticeable luminance dip in the filtered image of the stimulus (Westheimer, 1981), or when there is one unstimulated cone (or on-center ganglion cell) between two stimulated cones (this notion is often attributed to Helmholtz, 1909). Our result, in agreement with Toet et al. (1987) suggests that the human observer can do much better, in a signal-detection paradigm, when they are not restricted to judging that the lines appear separate. We find that two Gaussians separated by 0.9 SD can reliably be distinguished from a pair of Gaussians with no separation. This result is at variance with both the Rayleigh (1902) and Sparrow (1916) criteria for “resolution”. For example, the Gaussian equivalent to the Rayleigh criterion is shown in Fig. 5 (fourth from the top), where the separation is 2.79 SD, and the luminance dip is 26%. The Sparrow criterion (i.e. where the center of the distribution has a vanishing second derivative) is illustrated in Fig. 5 (third from the top), where the two Gaussians are separated by 2 SD. Our data show that “resolution” is compatible with the profile second from the top of Fig. 5, i.e. our observers could reliably distinguish ($d' = 1$) between two overlapped Gaussians (Fig. 5, top), and two Gaussians separated by about 1 SD. This result cannot be simply explained on the basis of luminance or contrast cues, since we jittered the contrast of the lines from trial to trial to render such information uninformative. Moreover, we obtained similar results when we increased the random contrast variation to 60%. One plausible explanation for these results is that 2-line “resolution” is in fact a size
discrimination, which is limited by the optical and/or neural blur of the visual system. A pair of Gaussians with \( \sigma = 1 \), separated by 1.0 \( \sigma \) is closely approximated by a single Gaussian (dotted line in Fig. 6) whose standard deviation is \( \sigma_1 + (1/2)^{0.5} \sigma_2 = 1.12 \), and whose amplitude is reduced by the same factor to keep the total area constant. For a “resolution” threshold of 0.9\( \sigma \), the change in standard deviation is about 10%. This width cue is distinguishable before the stimulus appears as two separate lines. Thus, while the “resolution” task is often thought of as discriminating two lines from one, in our task, the lines always appear single. It is tempting to speculate that in the “classical” “resolution” situation the observer may be using low frequency mechanisms to accomplish the “resolution”. Thus, although the two lines may appear doubled, the most sensitive cue might be the size cue. Future experiments in which the size cue is masked must be done to resolve this issue. For now we use the word “resolution” because our task is operationally the same as a resolution task. The key point is that this task is very sensitive to blur, so it is the appropriate task for measuring the limiting intrinsic blur.

Elsewhere, (Klein & Levi, 1989) we present a simple model in which the 10% width discrimination threshold is linked to the approx. 10% contrast discrimination threshold obtained under similar experimental conditions. The model’s prediction, with minimal assumptions, is that two Gaussians can be resolved when their separation equals about 1 SD. This is very close to our results, and it is tempting to speculate that this may provide a general principal for visual “resolution”. In peripheral vision, the variation in “resolution” for unblurred stimuli appears to reflect the limitations imposed by the variations in cone sampling, since in the central 10 deg, both vary with eccentricity with an \( E_2 \) of about 2–3 deg. Since the “resolution” threshold for unblurred stimuli is equal to the equivalent intrinsic blur, in the periphery, equivalent intrinsic blur is equal to optical blur plus sampling blur imposed by the spacing of cones.

What are the consequences of equivalent intrinsic blur for other tasks? Below we consider the role of equivalent intrinsic blur in detection of blurred lines, and in spatial interval discrimination.

**Experiment II: detection of Gaussian blurred lines**

Figure 6 shows detection thresholds plotted as a function of the standard deviation of the Gaussian blurred lines, at 0, 2.5, 5 and 10 deg in the lower visual field of observer DL (Fig. 6A) and WP (Fig. 6B). At each eccentricity, thresholds first decrease approximately linearly as the Gaussian standard deviation increases and then level off near 1%, almost independent of either width (Bijl, Koenderink & Toet, 1989) or eccentricity. The nonlinear regression curves fitted to the data have the form:

\[
T_\text{h} = T_\text{d} + B_\text{d} \quad \text{for } \sigma < B_\text{d}
\]

\[
T_\text{h} = T_\text{d} \quad \text{for } \sigma > B_\text{d}
\]

where \( \sigma \) is the standard deviation of the Gaussian stimulus blur, \( B_\text{d} \) is the critical stimulus blur value above which thresholds are independent of the standard deviation of the blur (we shall refer to this as the detection blur), and \( T_\text{d} \) is the asymptotic detection threshold. Of course, at yet larger standard deviations (greater than about 6 min in the fovea), thresholds

![Fig. 5. Illustrated here are pairs of Gaussians separated by 0 (top), 1 (2nd from top), 2.79 (4th from top) and 3.5 (bottom) standard deviations. Note that these separations correspond to those illustrated in Fig. 1. Our "resolution" data show that our observers could reliably distinguish (d' = 1) between two overlapped Gaussians (top), and two Gaussians separated by one (or 0.9 according to Fig. 4) standard deviation (2nd from top). Note that the latter curve is similar to a single Gaussian with a standard deviation \( \frac{1}{\sqrt{2}} \) times that of the single Gaussians comprising the pair, shown by the dotted line which has been displaced downward slightly for clarity.](image-url)
Figures 7A and B shows the detection blur ($B_d$) for each observer, plotted as a function of eccentricity. The left-hand ordinate shows the size of $B_d$ as the Gaussian standard deviation, while the right-hand ordinate shows the pooling area in minutes of rectangular blur (similar to Ricco's diameter). The critical blur increases approximately linearly with eccentricity. The eccentricity at which the foveal threshold doubles, $E_2$ (shown by the arrows in Fig. 7), was obtained for the data of each observer's critical blur, and these values and their error bars (obtained by nonlinear regression) are given in Table 1, along with $E_2$ values for several other tasks. For $B_d$, $E_2$ is between about 2.5 and 3.5 deg (this is discussed further below).

The data of Fig. 6 show clearly that for lines with a large blur (standard deviation $>$ 4 min), thresholds are almost independent of eccentricity up to an eccentricity of 10 deg. This can be seen more clearly in Fig. 7C, which shows the asymptotic detection thresholds ($T_a$) of the two observers plotted as a function of eccentricity. In contrast, for thin lines (e.g. less than 1 min), detection thresholds vary markedly with eccentricity. This is because, for stimuli smaller than the critical blur, visibility is equal to the product of contrast and width. Note that at each eccentricity in Fig. 6, thresholds for the smallest Gaussian standard deviation tested are approximately equal (about 4% for DL and 8% for HD). This corresponds to a line with a standard deviation 1 pixel wide at each eccentricity. Re-cause we "scaled" the stimulus size by varying the viewing distance with eccentricity, the angular subtense of the 1 pixel line increases with eccentricity; the similarity of the contrast thresholds with this SD = 1 pixel target, suggests that our choice of stimulus scaling factor (halving the viewing distance at 2.5 deg), essentially equated the visibility of the thin lines at each eccentricity. Moreover, as shown in Table 1, the value of $E_2$ for detecting a thin (rectangular) dark line is consistent with the scale factor.

The size of the detection blur ($B_d$), or Ricco's area, provides a measure of the distance over which the visual system pools contrast. A large number of studies have measured Ricco's area as a function of eccentricity. We found that Ricco's area (or $B_d$) doubled at about 2.5 deg. This is consistent with some previous studies (Scholtes & Boman, 1977; Webber, Manny, Levi & Klein, 1988); however, our $E_2$ value for spatial summation is considerably smaller than
those of several other investigators (e.g. Wilson, 1970; Ransom-Hogg & Spillman, 1980; see Table 1). We do not think that the discrepancy is a consequence of our use of Gaussian blurred lines (see below), but rather that it is a consequence of the temporal presentation. We used a slow onset and offset, with a long plateau, whereas all of the studies with shallower eccentricity dependence (large $E_2$ values), used brief, abrupt presentations. Transient temporal presentation is likely to degrade the foveal response, but have less effect upon the periphery. Interestingly, for such conditions, the $E_2$ is similar to that obtained for unreferenced motion (Levi et al., 1984).

In summary, we find that the linear pooling distance for spatial contrast (and presumably the size of the underlying detection mechanisms) is slightly over 1 min in the fovea (Gaussian SD of about 0.5 min), and varies with eccentricity with $E_2$ approx. 2.5 deg.

The relationship between Ricco’s extent ($B_d$) and “resolution” ($T_r$). The size of $B_d$ at each eccentricity is approximately equal to the unblurred “resolution” threshold $T_r$. This can be seen in Fig. 8, where we plot $B_d$ (the critical blur standard deviation) against the “resolution” threshold, $T_r$, for DL and WP at the fovea (○), and at 2.5, 5 and 10 deg in the lower visual field (●). The triangles show the foveal data of six other observers (the preferred eyes of amblyopes whose visual characteristics are shown in the subsequent paper). These data are included here to show the range of foveal resolution thresholds and of $B_d$. The diamonds and squares plotted along the lefthand axis of the graph show estimates of Ricco’s diameter (specified in terms of the standard deviation) from two other studies (Shapley, 1974, and Davila & Geisler, 1991, respectively). These data show a similar range of standard deviations ($B_d$) to the foveal data of the present study, and will be considered further in the Discussion section. The dashed line is the 1:1 line, and it is clear that most of the data fall close to but slightly above the line, suggesting that the same factor(s) constrain contrast pooling and “resolution”. We suggest that that factor is the equivalent intrinsic blur, $B_i$, which plays a dual role in determining both the “resolution” threshold, and the detection threshold. When the standard deviation of the stimulus blur, $\sigma$, is greater than $B_i$, the equivalent intrinsic blur, then the “resolution” threshold is approx. 0.9 $\sigma$ and the detection threshold is
Equivalent intrinsic blur

Fig. 8. The critical detection blur standard deviation ($B_d$ from Fig. 6) is plotted against the asymptotic (unblurred) “resolution” threshold for DL and WP (from Fig. 7) at the fovea ($O$), and at 2.5, 5 and 10 deg in the lower visual field ($O$). The triangles show the fovea data of six other observers (the preferred eyes of amblyopes whose visual characteristics are shown in the subsequent paper). These data are included here to show the range of fovea resolution thresholds and of $B_d$. The diamonds and squares along the lefthand axis of the graph show estimates of Ricco’s diameter (specified in terms of the standard deviation) from two other studies (Shapley, 1974, and Davila and Geisler, 1991, respectively). These data show a similar range of standard deviations ($B_d$) to the fovea data of the present study. The dashed line shown is the 1:1 line. The data fall close to the line, suggesting that the same mechanism(s) constrain contrast pooling and “resolution”.

approximately a fixed contrast ($T_r$). When the standard deviation of the stimulus blur, $\sigma$, is less than $B_r$, then the “resolution” threshold is approx. 0.9 $B_r$ and the detection threshold is approximately inversely proportional to $\sigma$ (it is $\approx T_r B_r/\sigma$), i.e. it obeys Ricco’s law. Comparison of Fig. 8 with Fig. 4 shows that $B_d$ is approximately equal to $B_r$. The summary in Table 3 of Levi and Klein (1990a) shows that the ratio $B_d/B_r = 0.96$ for the nonamblyopic eyes.

**Experiment III: effect of blur on spatial interval discrimination ($B_i$)**

Figures 9A and 10A show the effects of Gaussian blur on spatial interval discrimination at a number of base separations, for observers DL (Fig. 9A) and WP (Fig. 10A) in the fovea. Figure 9B shows the data of DL at 2.5 deg in the lower visual field; Fig. 10B shows data for WP at 10 deg. For each condition, there is a range of small stimulus blur values where the blur has little effect; at larger stimulus blur spread, thresholds rise in proportion to the stimulus blur. The most important point of note in Figs 9 and 10 is that the effect of stimulus blur depends upon the base separation of the lines. Consider the foveal data (Figs 9A and 10A). For line pairs with small separations, even a small amount of stimulus blur degrades threshold, whereas at large separations, spatial interval thresholds tolerate large degrees of stimulus blur.

The data in Figs 9 and 10 were fit according to:

$$T_h = k_r\sqrt{(\sigma^2 + B_i^2)}$$

(4)

$$= T_r\sqrt{(1 + \pi^2/B_r^2)}$$

(5)

where $T_r$ is the spatial interval discrimination threshold for an unblurred stimulus (i.e. the asymptotic threshold value), $\sigma$ is the stimulus blur (the abscissa), and $B_i$ is the “separation blur” (i.e. the horizontal position of the knee in the function). This is the critical blur standard deviation at which spatial interval discrimination thresholds are elevated by 40%. Note that points for $\sigma$ above 0.7 times the separation are excluded from the fitting, because when $\sigma$ equals the interline separation, the task becomes “resolution” which is a different task than interval discrimination of two separate lines. The separation blur ($B_i$), depends upon the base separation of the line pair. This dependence can be clearly seen in Fig. 11A, where we plot $B_i$ for the fovea of three observers (DL, WP and KP) as a function of separation. The dotted line has a slope of 1, and shows that for separations between about 1.4 and 14 min, the separation blur ($B_i$) is equal to approx. $1/4$ to $1/2$ of the separation between the lines. Thus, spatial interval discrimination is independent of stimulus blur, when the separation ($S$) is greater than about 3–4 times the standard deviation of the stimulus. For separation ($S$) much larger than the Gaussian standard deviation, threshold is proportional to separation (about $S/6$ of the separation). When the standard deviation of the blurred lines exceed about one-third of the line separation, separation thresholds are limited by the stimulus blur. In this regime thresholds never get smaller than about 0.2 $\sigma$. The average threshold is about 0.4 $\sigma$. The coefficient of $\sigma$ (0.4) is the multiplicative factor, $k_r$, in equation (3). Note that at the smallest separation, $B_i$ is about 0.3–0.4 min. Figure 11B shows $B_i$ of observers DL and WP at 2.5, 5 and 10 deg in the lower visual field over a range of separations.
Fig. 9. Line spatial interval discrimination thresholds as a function of the Gaussian spread of the stimulus (specified in terms of the Gaussian standard deviation) for observer DL. (A) Data are shown for observer DL's fovea for base separations of 1.4 (●), 2.5 (○), 5 (△), and 13 min (□). For comparison, the fit to his foveal "resolution" data are also shown by the dotted line. (B) Data are shown for observer DL at 2.5 deg in the lower visual field for base separations of 4.2 (●), 7.3 (○), 13 (△) and 26 min (△). For comparison, the fit to his "resolution" data at 2.5 deg are also shown by the dotted line. The lines fit to the data are of the form

\[ T_h = T_a [1 + (\sigma / B_t)^2]^{1/2}, \]

where \( T_a \) is the threshold for an unblurred stimulus (i.e., the asymptotic threshold value), \( \sigma \) is the stimulus blur, and \( B_t \) is the separation blur (i.e., the knee in the function).

The shaded region shows the range of foveal data from Fig. 11A. For all separations, \( B_t \) in the periphery is equal to approx. half of the interline separation. Thus, for separations less than about 20 min, the periphery appears to tolerate stimulus blur over slightly greater distances than in the fovea. Interestingly, for foveal viewing, at large separations, \( B_t \) is also about half of the interline separation. Thus, for separations greater than about 15-20 min, the eccentricity of the test lines may become an important factor (Klein & Levi, 1987).

Scale invariance? Figure 12 replots the spatial interval discrimination data of Figs 9 and 10, as well as similar data at other separations and eccentricities in a different format. In Fig. 12, both the thresholds and the stimulus blur have been divided by the interline separation. Thus, the thresholds are expressed as a Weber fraction \((\Delta S / S)\), and the stimulus blur standard deviation is also expressed as a fraction \((\sigma / S)\). In this figure each symbol represents a different eccentricity (○, fovea; △, 2.5 deg; □, 5 deg and ▲, 10 deg).
Fig. 11. (A) Separation blur ($B_s$) for the fovea of three observers, DL (□), WP (○) and KP (∆) as a function of separation. The dotted line has a slope of 1, and shows $B_s = 0.25 \times$ separation. For separations between about 1.4 and 14 min, the separation blur is between about $\frac{1}{2}$ and $\frac{1}{3}$ of the separation between the lines. In other words, for separations in this range, the visual system is tolerant to stimulus blur with a standard deviation of approx. 0.3 times the distance between the lines. For larger separations, the separation blur, $B_s$, is about half of the interline separation.

(B) Separation blur ($B_s$) of observers DL and WP at 2.5, 5 and 10 deg in the lower visual field over a range of separations. The shaded region shows the range of foveal data from (A). For all separations, the peripheral separation blur is equal to approx. half of the interline separation. Thus, for separations about 10 min or less, the periphery is tolerant to stimulus blur over greater distances than is the fovea.

Fig. 12. The spatial interval discrimination data of Figs 9 and 10, as well as similar data at other separations and eccentricities are replotted in a different format. Data are shown for observer DL in (A) and WP in (B). Both the thresholds and the stimulus blur have been divided by the interline separation. Thus, the thresholds are expressed as a Weber fraction ($\Delta S S$), and the stimulus blur standard deviation is also expressed as a fraction ($\sigma S$). In this figure, each symbol represents a different eccentricity (○, fovea; ▲, 2.5 deg; □, 5 deg and ●, 10 deg), and each symbol size represents a different separation. The smallest symbols, connected by lines, are the data near the optimal separation at each eccentricity. The most striking feature of this plot, is that much of the data, which spans about a 30-fold range of threshold and blur values when plotted in spatial coordinates, collapse into an almost unitary curve, with thresholds between about 0.06 and 0.12 for stimulus blur values less than about 0.3 (i.e. one-third of the interline separation), and rising sharply as the stimulus blur increases. At stimulus blur values near 1 (blur equal to separation), thresholds approach a Weber fraction of 1, i.e. the task becomes a "resolution" task. The data at the smallest separations (near the optimal separation) at each peripheral locus are represented by the small symbols connected by dotted and dashed lines, and they differ in two main respects from the foveal data (○): (i) at small stimulus blur values the peripheral threshold Weber fractions are 2–3 times higher than the foveal values; and (ii) the peripheral thresholds begin to rise when the stimulus blur exceeds about 0.5 (i.e. half the interline separation). Note that when the stimulus blur exceeds about 0.5, thresholds at all eccentricities and separations converge.
with thresholds between about 0.06 and 0.12 for stimulus blurs less than about 0.3 (i.e. one-third of the interline separation), and rise sharply as the stimulus blur increases. At stimulus blur values near 1 (blur equal to separation), thresholds approach a Weber fraction of 1, i.e. the task becomes a “resolution” task. Although much of the data, when plotted in this fashion appears to be scale invariant (Toet et al., 1987, 1988), the data at the smallest separations (near the optimal separation) at each peripheral locus stand out. These data are represented by the small symbols connected by dotted and dashed lines, and they differ in two main respects from the foveal data (○): (i) at small stimulus blur values the peripheral threshold Weber fractions are 2–3 times higher than the foveal values; and (ii) the peripheral thresholds begin to rise when the stimulus blur exceeds about 0.5 (i.e. half the interline separation). Note than when the stimulus blur exceeds about 0.5, thresholds at all eccentricities and separations converge. Elsewhere (Klein & Levi, 1989), we show that a simple local contrast model predicts that blur should degrade spatial interval discrimination when the blur standard deviation exceeds approx. one-third of the separation. The apparent tolerance of the periphery to blur near the optimal separation, is a consequence of raised positional uncertainty, which elevates the optimal threshold in the periphery. Thus, blur appears to have less effect, because the peripheral performance is already degraded.

Experiment IV: “resolution” and spatial interval discrimination with unblurred lines

In these experiments, we measured 2-line spatial interval discrimination thresholds using thin (unblurred rectangular) dark lines over a range of base separations. Figure 13 shows spatial interval discrimination thresholds plotted against the base separation of the line pair, at 0, 2.5, 5 and 10 deg in the lower visual field. The contrast of the lines was 5 times the contrast threshold for detecting a single line, at each eccentricity. Results are shown for DL (Fig. 13A) and HD (Fig. 13B). First consider the foveal thresholds, shown by the open circles. The leftmost point (at each eccentricity) represents the 2-line “resolution” limit, i.e. the smallest separation that could be reliably distinguished from a pair of overlapped lines. The thresholds of between 0.35 and 0.4 min are quite small considering the stimuli are pairs of lines with each line at five times its detection threshold. At higher contrasts the “resolution” thresholds are even smaller, so they become hyperacuities (Carney et al., 1988). The position of the “resolution” datum on the horizontal axis is chosen to be equal to half the “resolution” threshold. For separations greater than the “resolution” limit, thresholds improve dramatically, reaching an optimum of around 4 sec arc at about a 1 min separation. Note that in the fovea, the transition from “resolution” to the optimal hyperacuity is very steep. For separations larger than the optimum, thresholds rise in proportion to the line separation. This “Weber’s Law” for spatial interval discrimination has been frequently described (Fechner, 1860; Westheimer & McKee, 1977; Hirsch & Hylton, 1982; Levi, Klein & Yap, 1988). For our experimental conditions (low contrast lines on a uniform background), the Weber fraction for separations between about 1 and 15 min is about 1/16, as shown by the dotted line. At larger separations, where the eccentricity of the target lines becomes important, the

In the periphery, the functions in Fig. 13 differ in three ways from those of the fovea: (1) the "resolution" limit is larger; (2) the transition from "resolution" to the optimal spatial interval discrimination is less steep than in the fovea; and (3) the Weber fraction $\Delta S/S$ is slightly worse than that of the fovea.

Figure 14 plots the "resolution" limit (○) and optimal spatial interval discrimination threshold (□) as a function of eccentricity for DL (A) and HD (B). The lines were fit by linear regression to the data, and the values of $E_2$ for "resolution" and for the optimal spatial interval discrimination threshold estimated from the fit are presented in Table 1. Note that because the data are plotted in absolute (rather than relative units), the slopes of the lines do not show the rate of fall-off with eccentricity relative to the fovea. Rather, it is the $X$-axis intercepts of the lines (denoted by $E_2$, i.e. the doubling eccentricity) which illustrate the relative rate of fall-off of periphery vs fovea. For each observer the value of $E_2$ for "resolution" is about 3–4 times larger than that for spatial interval discrimination. For "resolution", the $E_2$ values were between 2 and 2.5 deg, similar to the variation in cone spacing with eccentricity (Klein & Levi, 1987). The solid triangles at 0, 3.8 and 10 deg eccentricity in Fig. 14A represent a psychophysical estimate of DL's cone spacing in his lower visual field. These data were obtained in David Williams' laboratory using the motion reversal technique of Coletta and Williams (1987a, b), and are labelled "sampling limit" in the figure. While the cone separation data at 3.8 deg seems large compared to the "resolution" thresholds, the correspondence between these data and the "resolution" thresholds at 0 and 10 deg is remarkable. Note, however, that the 2-line "resolution" results are not compatible with classical notions of visual acuity. For example, one theory often attributed to Helmholtz (1909), suggests that at the "resolution" limit, there should be one unstimulated cone between two stimulated ones, i.e. "resolution" should be equal to twice the cone spacing. In more modern terms, this is known as the sampling theorem. Our results suggest that 2-line "resolution" is approximately equal to cone spacing. We argue below that while cone spacing may ultimately limit 2-line "resolution", there is a more general explanation (equivalent intrinsic blur) for 2-line "resolution" thresholds (see the section on blur below). It is also worth noting that the nice agreement between "resolution" and the sampling limit, probably does not hold at eccentricities beyond about 10 deg, because of pooling by ganglion cells (Wassle, Grunert, Rohrenbeck & Boycott, 1989; Thibos, Cheney & Walsh, 1987).

The values of $E_2$ for the optimal spatial interval discrimination thresholds were 0.68 ± 0.08 deg and 0.83 ± 0.1 deg, for DL and HD respectively. Also given in Table 1 is the $E_2$ values for a third observer, WP. His $E_2$ value was 0.78 ± 0.09 deg. These values are well within the range of values, between about 0.4 and 0.9 deg, reported for other positional acuity.
tasks (Westheimer, 1982; Levi et al., 1985; Yap, Levi & Klein, 1987; Klein & Levi, 1987; Yap, Levi & Klein, 1989), and are consistent with several recent estimates of the variation in the cortical sampling grain in monkey VI (Dow, Snyder, Vautin & Bauer, 1981; Dow, Vautin & Bauer, 1985; Tootell, Silverman, Switkes & De Valois, 1982; Van Essen, Newsome & Maunsell, 1984; Tolhurst & Ling, 1988), i.e. both the optimal threshold and the cortical sampling grain change by about a factor of 16 between 0 and 10 deg. Note however, that Schwartz (personal communication) argues for a value of $E_2$ closer to 0.3 deg on the basis of 2DG studies, while Tootell, Switkes, Silverman and Hamilton (1988), argue for a somewhat larger value based on similar studies. The precise value of $E_2$ for cortical magnification, or for ganglion cell sampling (Wassle et al., 1989) remains somewhat uncertain, because it depends critically upon a precise measure of the foveal value. It is less of an issue for psychophysics, where the foveal value can be specified very precisely. Interestingly, the separation yielding the optimal threshold (the optimal separation—shown by the diamonds in Fig. 14) varies slowly with eccentricity. For example for observer DL, the optimal separation is just under 1 min in the fovea, and is about 5 min at 10 deg. The mean value of $E_2$ for the optimal separation for the three observers was 2 deg, consistent with the variation in "resolution" and equivalent intrinsic blur, rather than with the variation in optimal thresholds (Table 1).

SUMMARY AND GENERAL DISCUSSION

To summarize, we used Gaussian blurred stimuli to measure visual performance for "resolution": detection and spatial interval discrimination.

Our "resolution" experiments show that 2-line "resolution" thresholds are between 0.35 and 0.6 min in the fovea, and are degraded when the stimulus blur standard deviation exceeds approx. 0.5 min. At each eccentricity our results are compatible with the formula:

$$Th = k\sqrt{(\sigma^2 + B_i^2)}; \quad (1)$$

where $k \approx 0.9$ and $B_i$ is an internal error in the visual system that acts like blur. $B_i$ reflects the amount of stimulus blur that raises threshold by 40%, and we have shown that it is largely independent of stimulus contrast. As will be discussed in the next paper, intrinsic blur, $B_i$, is not always contrast independent. In anisometropic amblyopes, $B_i$ decreases as contrast increases. In normal foveal and peripheral vision, however, $B_i$ does not depend on contrast. The equivalent intrinsic blur standard deviation doubles at about 2–3 deg in the periphery. Watt and Morgan (1984) and more recently, Watt and Hess (1987) suggested that the visual system has an internal error that acts like blur. If the stimulus blur is smaller than this internal error, it will have no effect upon performance; however, when it exceeds the internal error, then threshold will be proportional to the stimulus blur. The size of the equivalent intrinsic blur reflects the magnitude of this internal error, and we suggest that it reflects the cascaded effects of both optical and neural filtering by the visual nervous system and the discrete spatial sampling by cones. When the optical blur becomes small compared to the spacing of the sampling mosaic, as occurs in the periphery, then the performance must be governed by the spacing.

Our results suggest that in both central and peripheral vision there is a close connection between equivalent intrinsic blur, "resolution" and Ricco's diameter. We find that:

(i) When the standard deviation of the stimulus blur, $\sigma$, is greater than $B_i$, the equivalent intrinsic blur, then the "resolution" threshold is approximately equal to $\sigma$ and the detection threshold is approximately a fixed contrast (to be referred to as $T_d$).

(ii) When the standard deviation of the stimulus blur, $\sigma$, is less than $B_i$, then the "resolution" threshold is approximately equal to $B_i$ and the detection threshold is approximately inversely proportional to $\sigma$ (it is $\approx T_d B_i/\sigma$), i.e. it obeys Ricco's law.

According to (i) and (ii), the equivalent intrinsic blur $B_i$ plays a dual role in determining both the "resolution" threshold and the detection threshold. $B_i$ corresponds to the "Ricco's diameter" for spatial summation in a detection task, and it also corresponds to the "resolution" threshold for thin lines. This connection between detection and "resolution" is somewhat surprising because many (but by no means all) previous estimates of Ricco's area are considerably larger than ours.

In comparing our estimate of Ricco's diameter $\sqrt{(2\pi)}(B_i \approx 0.5 \text{ min})$ to previous estimates, it should be noted that (a) we specify Ricco's diameter as the standard deviation of the stimu-
lus blur. It should be multiplied by $\sqrt{(2\pi)}$ in order to compare with the more standard definition of the full width of an equal area rectangular profile. (b) our experiments were performed at a high photopic luminance, and with slow pattern onset and offset in order to minimize temporal transients, (c) our stimulus size was varied in only one dimension, whereas many classical studies used spots which varied in two dimensions. However, our data, and the relationship between Ricco’s diameter and “resolution” are consistent with the data of Webber et al. (1988) using very different stimuli and methods, and with both the data and the ideal-observer computations of Davila and Geisler (1987, 1991). The most closely related study of summation using sustained presentation of Gaussian bars at a high photopic luminance level, is that of Shapley (1974), so it is of considerable interest to compare his data with ours. Unfortunately, Shapley did not test at very small bar widths, so he did not have data in the Ricco’s law regime of full summation. His data has a slope of about $-0.5$ (Piper’s law) rather than $-1$ (Rico’s law). We choose to define Rico’s diameter in terms of the region of total summation rather than partial summation (Piper’s regime). Partial summation might be due to probability summation that covers an extent larger than the intrinsic blur, so the summation extent would be delicately depending upon assumptions about the exact nature of the summation region. Gorea and Tyler (1986) examined the temporal equivalent of Rico’s area (Bloch’s Law), and showed that different assumptions about probability summation led to different values of the summation extent. In our prescription, Rico’s diameter is given by the horizontal position of the intersection of two lines: (1) a line of slope $-1$ (on a plot of log threshold vs log blur) through the datum with the smallest blur. This is the line of total summation; (2) a line of slope 0 (a flat line) through the optimal (lowest) threshold of the Gaussian bar. This is the line of maximum sensitivity. This is similar to the approach of Webber et al. (1988), i.e. taking the ratio of the detection thresholds for a thin line and an edge. In order to calculate Rico’s diameter from Shapley’s data, we first determine the line moment of the thinnest line that was used. The line moment is the area under the profile and is given by the contrast times $\sqrt{(2\pi)}$ times $\sigma$ for a Gaussian. The units are % min, representing the contrast in % times the width in minutes. We then determine the detection threshold in % contrast of the Gaussian bar that has the lowest threshold. The Rico diameter (min) is then determined by dividing the line moment by the most sensitive detection contrast. The % contrast in the numerator and denominator cancel. Shapley’s contrast threshold for his narrowest bar width is approx. 2% at an “effective width” of about 0.011 deg or about 0.67 min. This leads to a line threshold of 1.3% min (in agreement with Hecht & Mintz, 1939). Shapley’s lowest detection threshold is approx. 0.5%, so his Rico’s diameter is about $1.3/0.5 = 2.6$ min or a SD of 1.04 min. This is slightly larger than the average of $0.72 \pm 0.18$ min in our observers (see Fig. 8, and also Table 2 of Levi & Klein, 1990a). However, both Shapley’s data and our own are in surprisingly close agreement with the data of Davila and Geisler (1991), and all are shown in Fig. 8. Despite their lower luminance level and the use of two-dimensional stimuli (both of which might be expected to produce larger summation areas), Davila and Geisler’s observers showed a range of Rico’s diameters (specified in terms of the standard deviation) from 0.66 to 1.06 min.

Our “resolution” thresholds $(T, \approx 0.9 B, \text{ at low contrasts})$ are also smaller than most previous estimates $(T, \approx 2 B,)$. For high contrasts, resolution thresholds of 0.33 min are found (Carney et al., 1988), compatible with the present “resolution” findings of 0.35 and 0.4 min shown in Fig. 13 for stimuli consisting of lines at five times detection threshold. An important factor in these small thresholds is that we used a criterion-free signal detection method with feedback. For very small separations, the observer could use a width (size) cue, since for small separations, increasing the line separation is equivalent to increasing the standard deviation of a single Gaussian. At the “resolution” threshold, this is equivalent to a change in standard deviation of about 12%. This is shown by the dotted line in Fig. 5, which is remarkably similar to the just resolvable pair of smaller Gaussians. This width cue was distinguishable before the stimulus appeared as two separate lines.

Both the unblurred “resolution” threshold and the equivalent intrinsic blur, $B$, vary with eccentricity in a manner consistent with the variation of cone separation, and with the size of cortical receptive fields. When the stimulus blur exceeds the equivalent intrinsic blur, “resolution” in the periphery is the same as in the
fovea. If our hypothesis that 2-line "resolution" is based upon width discrimination is correct, then our results imply that width judgements are good in the periphery for blurred stimuli (Fig. 2), but not for sharp stimuli (Fig. 13).

(iii) Spatial interval discrimination thresholds are proportional to the separation of the lines (i.e. Weber's law). At the optimal separation, spatial interval discrimination thresholds represent a "hyperacuity" (i.e. they are smaller than the "resolution" threshold). For unblurred lines, the optimal separation is 2-3 times the "resolution" limit at all eccentricities, so the optimal separation varies at the same rate as \( B_t \). However, the optimal spatial interval threshold falls off about 3-4 times more rapidly, consistent with the rate of decline of other position acuity tasks. For Gaussian blurred lines, over a wide range of separations and eccentricities, spatial interval discrimination thresholds begin to rise when the stimulus blur exceeds between about \( \frac{1}{3} \) to \( \frac{1}{2} \) the separation of the lines (see Fig. 12). The strong elevation of the optimal spatial interval discrimination threshold in the periphery cannot be predicted on the basis of the detectability of the lines, "resolution", or on the basis of equivalent intrinsic blur. We hypothesize that the increased spatial interval discrimination thresholds are a consequence of position uncertainty due to sparse spatial sampling in the periphery. Below we examine in more detail the nature of the equivalent intrinsic blur, and its relationship to spatial filtering, spatial sampling, and positional uncertainty.

The effect of blur on "resolution"

The present results suggest that 2-line "resolution" is equal to the larger of either \( B_t \), or the standard deviation of the externally imposed stimulus blur. The latter alternative is consistent with the recent studies of Toet and Koenderink (1987), and suggests that under our conditions 2-line "resolution" may be a form of shape or size discrimination. In central vision, blurring by the eye's optics and by the visual nervous system are rather well matched (Westheimer, 1981); however, in peripheral vision (at least within the central 10 deg), there is little change in the optics of the eye (Jennings & Charman, 1981), so that variations in the equivalent intrinsic blur of the visual system must reflect neural changes with eccentricity. Indeed, both the "resolution" limit, and the equivalent intrinsic blur grow with eccentricity at a rate consistent with the variation in cone spacing within the central 10 deg. Beyond about 10 deg this nice correspondence would be expected to break down due to ganglion cell pooling (Wassle et al., 1989).

The effect of blur on spatial interval discrimination

A striking result of this study is that \( B_t \) (the critical blur standard deviation above which spatial interval discrimination is degraded) grows approximately proportionally with the separation between the lines, i.e. the visual system's tolerance to stimulus blur increases with line separation. In foveal vision, when the standard deviation of the blur exceeds about one-third of the line separation (S), thresholds are degraded. Over a wide range of separations and eccentricities, the effects of blur on spatial interval discrimination thresholds are approximately scale invariant (see Fig. 12). At all levels of blur, and at each eccentricity, at the optimal separation, 2-line interval discrimination represents a "hyperacuity" (i.e. interval thresholds are smaller than the "resolution" limit), and the optimal separation is 2-3 times the "resolution" limit. For unblurred lines, the optimal separation is approx. 2-3 times \( B_t \).

Snyder (1982) suggested that blurring the stimulus may actually improve positional acuity in peripheral vision, where spatial sampling is sparse. Our results with long lines, and long viewing duration do not support his hypothesis, because there is no dipper in the data of Figs 9, 10 or 12 that would indicate facilitation due to blur in the periphery. The lack of facilitation may be because our stimuli already provide sufficient samples in time and space. Our previous work (Levi & Klein, 1986; Levi, Klein & Yap, 1987) suggests that with very small, brief stimuli, spatial interval discrimination in the periphery improves in proportion to the square root of the number of discrete samples along the length (but not the breadth) of the lines.

Scaling of peripheral function

Two-line spatial interval discrimination is a one-dimensional task which places "resolution" and spatial interval discrimination (a hyperacuity) on a continuum. Our results suggest that two factors are needed to account for the variation in thresholds with eccentricity: (1) a scale factor with \( E_2 \) of about 2-3 deg is needed to account for the variation of "resolution"
thresholds, $T_e$, the intrinsic blur, $B_i$, and the position of the optimal separation (i.e. the separation yielding the best threshold at each eccentricity). This scale factor is consistent with the variation in cone separation (Osterberg, 1955; Curcio et al., 1987; Coletta & Williams, 1987a, b), and with the variation in cortical receptive field size (Dow et al., 1981); (2) a second scale factor is needed, with $E_2 = 0.4-0.9$ deg, to equate the optimal spatial interval discrimination thresholds $T_e$. This value of $E_2$ is consistent with previous estimates of the variation in position thresholds with eccentricity (Westheimer, 1982; Levi et al., 1985; Klein & Levi, 1987; Yap et al., 1987, 1989), and is also consistent with several recent estimates of the variation in the cortical sampling grain in monkey (Dow et al., 1981, 1985; Tootell et al., 1982; Van Essen et al., 1984) and human (Tolhurst & Ling, 1988) V1. [Note however, Schwartz (personal communication) argues on the basis of 2-DG data that 0.3 deg is a better estimate, while Tootell et al. (1988) use similar data to argue for a somewhat larger value.] The present 2-line data are in agreement with the two-dot “resolution” data of Yap et al. (1989), and the two-dot Vernier acuity data of Westheimer (1982). Yap et al. (1989) argued that the failure of a single scale factor in these tasks reflects different limitations imposed by the visual nervous system. At very small separations, close to the “resolution” limit, optical and neural blur may limit performance, whereas at slightly larger separations (eccentricities), hyperacuity thresholds may be limited by the spatial sampling grain ganglion cell or cortical.

In the present study we measured the equivalent intrinsic blur of the visual system. We found that both the equivalent intrinsic blur ($B_i$) and the optimal separation (the separation yielding the lowest thresholds) varied with eccentricity with an $E_2$ of about 2 deg. Recently, Hess, Pointer and Watt (1989) found similar results on a different task. They found that the optimal edge blur discrimination threshold increased between 5 and 7 fold from the fovea to 2.5 deg ($E_2$ of about 0.4–0.6 deg), while the position of the optimum (i.e. the blur pedestal) increased at a much slower rate ($E_2$ of about 2 deg).

In summary, in the present study we measured the equivalent intrinsic blur of the visual system. We found that in the fovea it has a standard deviation of approx. 0.5 min, and that it doubles at about 2–3 deg in the periphery. Equivalent intrinsic blur appears to play an important role in normal spatial vision, determining both the “resolution” threshold and the detection threshold in central and peripheral vision. Equivalent intrinsic blur may also play an important role in limiting the optimal separation for spatial interval discrimination in the normal fovea. In the periphery, the strong elevation of the optimal spatial interval discrimination threshold cannot be fully accounted for on the basis of the detectability of the lines, their “resolution”, or on the basis of equivalent intrinsic blur. We hypothesize that the increased spatial interval discrimination thresholds are a consequence of position uncertainty, perhaps due to sparse spatial sampling in the periphery.

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